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Water resources planning and management

by use of generalized Benders decomposition method to solve large-scale MINLP problems

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Contents

- Characteristic features of water resources systems: large-scale systems, particular structures, single/ multiple objectives, real and discrete decision variables, fuzzy/ nonfuzzy environment, inequality/ equality constraints, bounded variables, nonlinearities, discontinuities, nonconvexities, nonlinear dynamics.
- Optimization routines for large-scale simulation systems: optimization routines in simulation models, mixed-integer nonlinear programming, decomposition methods, branch-and-bound techniques, generalized Benders decomposition, flowchart of the algorithm with illustrative solved numerical example, software packages.
- Water planning and management problems: water resources systems planning and management, groundwater management, water quality management. Small-size illustrative examples, main features of recent applications to WRS planning and management, groundwater management, and water quality management.

references

●[1] A.A. Keller, ''Multiple-use water resources management by using fuzzy multi-objective heuristic optimization methods: an overview'', *International Journal of Fuzzy Systems and Advanced Applications*, **1** (2014) 36-54.

●[2] A.A. Keller, ''Water resources planning and management by use of generalized Benders decomposition method to solve large-scale MINLP problems'', *Journal of Water Resource and Hydraulic Engineering* **4**, 1-4 (2015) 136-148.

[3] A.A. Keller, ''Convex underestimating relaxation techniques for nonconvex polynomial programming problems: computational overview'', *Journal of the Mechanical Behavior of Materials* (De Gruyter, Berlin), in Press, 2015.

●[4] A.A. Keller, ''Evolutionary Multi-Objective Optimization: Theory and Applications'', Bentham Science eBooks (in preparation: 2015).

1. Main characteristic features of water resources systems

►**Large-sale system** due to 1) a large number of decision variables; 2) a combinatorial number of alternative solutions.
►**Mixed-integer bounded decision variables**: e.g. in groundwater management, pumping rates are continuous decision variables and locations of wells are integer decision variables.
►**Constrained optimization** with single/ multiple objective(s), equality and inequality constraints: 1) physical objectives, e.g. maximizing irrigation releases, hydropower production, groundwater quality; 2) economic objectives, e.g. maximizing net returns, minimizing costs, investment in water development; 3) physical and economic constraints, e;g. reservoir storage, surface water balance, water supply, demand and costs.
►**Uncertainties** related to: 1) variability of climate and hydrology e.g. flow streams, rainfalls and temperatures; 2) water quality e.g. random pollution transport; 3) imprecision and lack of data; 4) environmental policy, vagueness of judgment from decision makers (fuzzy character of data and relations) .

2. Combined algorithms for solving large-scale mixed-integer programming problems

1. Decomposition-based algorithm (GBD)
Principle: The procedure consists of a master program with interacting sub-problems. 1) Sub-problems receive a set of simplex multipliers and apply their solutions to the master program. 2) The master problem combines this information to compute new duals.
2. Branch-and-bound enumerative algorithm (B&B)
Principle: The procedure consists of two basic operations. 1) at the branching step the feasible space is partiionned in smaller subsets; 2) at the bounding step a lower bound (for a minimization problem) is calculated for each subset.

►Mixed-integer nonlinear programming

Single objective mixed-integer nonlinear programming (MINLP) [1]	Multiobjective mixed-integer nonlinear programming (pMINLP) [2-3]
Problem: Minimize one objective function subject to equality and inequality constraints and bounds with real and integer design variables.	**Problem**: Minimize multiple objective functions simultaneously subject to constraints and bounds with real and integer design variables.
Method: Iterative decomposition-based algorithm where sub-problems interact with a master problem.	**Method:** Iterative decomposition-based algorithm where sub-problems interact with a master problem.
Formulation: MINLP problem, ILP (master problem), NLP (sub-problems)	**Formulation**: pMINLP (parametric) problem.
Solution: Global optimum	**Solution**: Noninferior solution set

►GBD algorithm:

● GBD generates an upper bound and a lower bound of the approximated solution at each iteration.

● It consists of a sequence of NLP sub-problems and of a master ILP problem. NLPs provide upper bounds whereas mater IP yields a lower bound to the optimal solution.

● A primal sub-problem corresponds to the original problem with fixed discrete variables. It provides information about the upper bounds and Lagrange multipliers. The master problem is derived from nonlinear duality theory and calculates the next set of lower bounds.

● The sequence of updated upper bounds is nonincreasing and that of lower bounds is nondecreasing. A convergence is attained after a finite number of iterations.

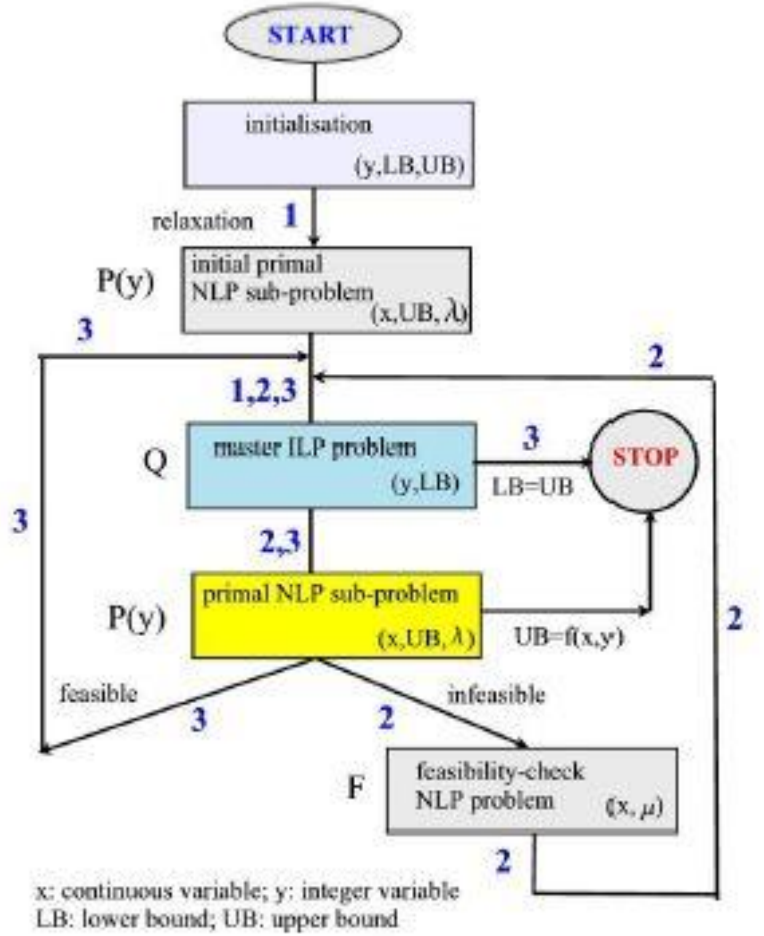

Fig. 2 Flow chart of the GBD algorithm in the illustrative example.

►Illustrative example

(inspired from Li & Sun, 2006)

$$\underset{x\in X,y\in Y}{\text{minimize}} \quad f(x,y) = -2\ln(1+x)+5y$$

$$\text{subject to : 1) } g_1(x,y) = -1+\exp\left(\frac{x}{2}\right)-\frac{\sqrt{y}}{2} \le 0,$$

$$\text{2) } g_2(x,y) = 2.5-2\ln(1+x)-y \le 0,$$

$$\text{3) } g_3(x,y) = -4+x+y \le 0,$$

$$\text{4) } x \in X = [0,2],\ y \in Y = \{1,2,3\},$$

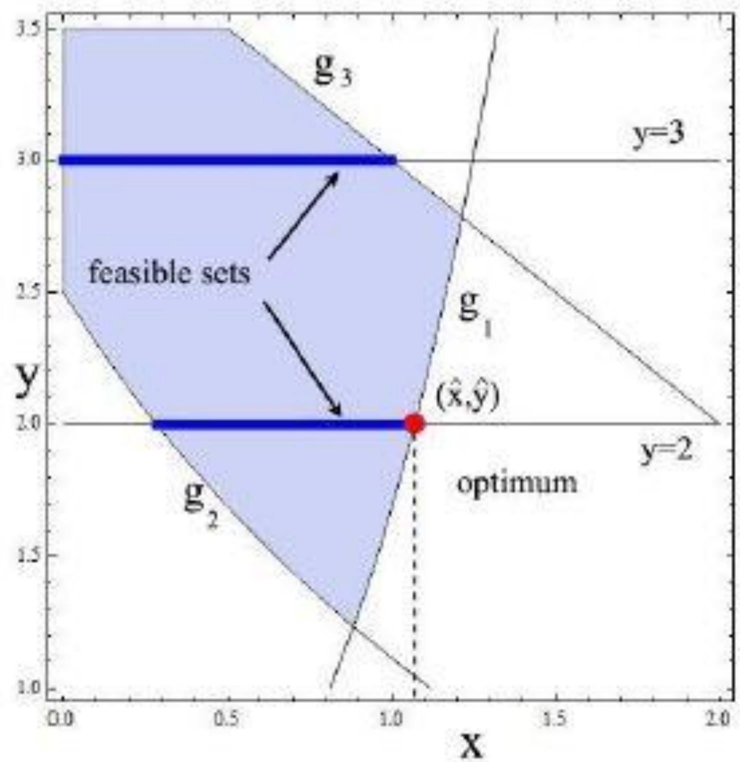

Fig. 1. Feasible sets and solution of the illustrative example.

3. Real-life application to WRSs

Table 1 WATER RESOURCE MANAGEMENT APPLICATIONS

#	Type of problem	Programming model description	Type of method	Reference
► WATER RESOURCES SYSTEMS PLANNING & MANAGEMENT				
1	•Planning with shortage control (e.g., dimension of water works, reliability of the system). •Management (e.g., vulnerability of the system to water resource shortage risk). •Maintenance of pipe-networks for water supply.	•**Decision variables**: Time independent planning variables (e.g., storage volume of reservoirs, extension of irrigation sites, etc.); time dependent operating variables to meet system requirements. •**Objectives**: To minimize costs of constructions, maintenance, and operating costs. •**Constraints**: Mass balance; flow continuity equations; links between planning constraints, filling capacity of reservoirs.	•Multi-period network; optimization model.	Liberatore et al. (2006)[23]
2	•Reservoir system operation for water supply and power generation. •River basin water allocation. •Control and salinity.	•**Objectives**: To maximize a weighted combination of objectives: the ratio of delivered water to demand; the smallest water deficit among all sites, the total amount of generated hydropower in the basin, to minimize salt concentration. •**Constraints**: Water balance at river, at reservoir nodes, at groundwater, etc.; power generation, salinity balance at river, canal.	•Model network, large-scale nonconvex NLP, generalized Bender decomposition algorithm, relaxing the constraints. •Two models: 1) reservoir operation model; 2) water basin allocation and salinity control model.	Cai et al. (2001)[24]
► GROUNDWATER MANAGEMENT				
3	•Design and water management. •Least-cost conjunctive use strategies for managing surface water and groundwater resource systems.	•**Decision variables**: Continuous variables (i.e., extraction, injection rates from wells); discrete variables (i.e., well locations). •**Objectives**: To minimize operation costs. •**Constraints**: Desired groundwater levels; pumping capacities; meeting water demands.	•MINLP model. •Solved by branch-and-bound method, GBD algorithm, global search techniques.	Chiu et al. (2012)[2]

Thank you for your attention !