Fuzzy Logic Games in Economics

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In the real game situations, the possible values of parameters are imprecisely known to the experts and all data of the game are not exactly known by players. Imprecision on the environment, preferences, payoffs and moves of other players,may be of different types, but not only the probabilistic type of the Bayesian games. Researchs on fuzzy games have been developed rapidly since the mid 1970s [NeRa75][Bu78][Bu79][NiSa01][Zi01]. Nishizaki and Sakawa [NiSa00b] formulated an LP problem with fuzzy (triangular) parameters and a fuzzy goal of each coalition of players. In the fuzzy programming problem, the decision maker may know the costs of the objective function, whereas the payoffs in the constraints would stay imprecise [Ca89]. The resolution method consists in introducing tolerance levels for the violation of each constraint [DeVeVi89]. The concepts of equilibrium may be based on Zimmermann's approach, in two steps for solving linear multi-objective problems with fuzzy goals [BeZa70][Ch83][KiLe01][BeCh05][KaLa08]. Fuzzy logic games (FLG) are a component of a larger class of combinatorial games and also belong to the so-called "soft computing" which combines fuzzy logic, neural networks and evolutionary programming [Au81][Au82]. In FLGs, the decision making is re-formulated in an uncertain (fuzzy) environment : the decision makers are confronted with fuzzy constraints, fuzzy utility maximization and also fuzziness about the moves of the competitors [Wi04][Ca89]. Moreover, cooperative FLGs are describing coalitions, where the n players associate a certain rate to their participation. Such games are defined on fuzzy subsets of the whole set of n players [Au81][Au82][GaCr03][**?**][Hw07]. Billot [Bi92] reformulated the basic microeconomic theory to deal with fuzzy choice and preferences. The fuzzy preference operator $\succsim$ is defined by $\succsim: X^2 \mapsto [0,1]$, X denoting the set of alternatives. Hence $x \succsim y$ will show the degree to which x is at least as good as y. This contribution is proposing a survey on these techniques with numerical applications to economics. The computations are carried out using the software $MATHEMATICA^{®}$ 7.

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