GAME THEORY AND MANAGEMENT. Collected abstracts of papers presented on the Second International Conference Game Theory and Management

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THE INTERNATIONAL SOCIETY OF DYNAMIC GAMES
(Russian Chapter)

GAME THEORY AND MANAGEMENT

The Second International Conference
Game Theory and Management

GTM2008

June 26-27, 2008, St. Petersburg, Russia

ABSTRACTS

Edited by Leon A. Petrosjan and Nikolay A. Zenkevich

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The collection contains abstracts of papers accepted for the International Conference Game Theory and Management (June 26–27, 2008, Saint – Petersburg University, Saint Petersburg, Russia). The presented abstracts belong to the field of game theory and its applications to management.

The abstract volume may be recommended for researches and post-graduate students of management, economic and applied mathematics departments.

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ТЕОРИЯ ИГР И МЕНЕДЖМЕНТ. Сб. тезисов 2-ой международной конференции по теории игр и менеджменту / Под ред. Л. А. Петросяна, Н. А. Зенкевича. – СПб.: Высшая школа менеджмента СПбГУ, 2008. – 236 с.

Сборник содержит тезисы докладов участников 2-ой международной конференции Теория Игр и Менеджмент 2008 (26–27 июня 2008 года, Высшая школа менеджмента, Санкт-Петербургский государственный университет, Санкт-Петербург, Россия). Представленные тезисы относятся к теории игр и её приложениям в менеджменте.

Тезисы представляют интерес для научных работников, аспирантов и студентов старших курсов университетов, специализирующихся по менеджменту, экономике и прикладной математике.

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WELCOME ADDRESS

We are very pleased to welcome you at the Second International Conference on *Game Theory and Management* (GTM2008) which has been organized by the Graduate School of Management (GSOM), Saint Petersburg State University (SPbSU) in collaboration with the Faculty of Applied Mathematics & Control Processes, SPbSU and with the International Society of Dynamic Games (Russian Chapter).

The Conference is designed to support further development of dialogue between fundamental game theory research and advanced studies in management. Such collaboration had already proved to be very fruitful, and has been manifested in the last two decades by Nobel Prizes in Economics awarded to John Nash, Robert Aumann, John Harsanyi and few other leading scholars in game theory. In its applications to management topics game theory contributed in very significant ways to enhancement of our understanding of the most complex issues in competitive strategy, industrial organization and operations management, to name a few areas.

Needless to say that *Game Theory and Management* is a very natural area to be developed in the multidisciplinary environment of St. Petersburg University which is the oldest (est. 1724) Russian classical research University. This Conference was initiated in 2006 at SPbSU as part of the strategic partnership of its GSOM and the Faculty of Applied Mathematics & Control Processes, both internationally recognized centers of research and teaching.

We would like to express our gratitude to the Conference's key speakers – distinguished scholars with path-breaking contributions to economic theory, game theory and management – for accepting our invitations. We would also like to thank all the participants who have generously provided their research papers for this important event. We are pleased that this Conference has already become a tradition and wish all the success and solid worldwide recognition.

Co-Chairs GTM2008

Professor Valery S. Katkalo	Professor Leon A. Petrosyan
Dean, Graduate School of Management	Dean, Faculty of Applied Mathematics & Control Processes

St. Petersburg State University

WELCOME

On behalf of the Organizing and Program Committee of GTM2008, it gives us much pleasure to welcome you to the International Conference on Game Theory and Management in the Graduate School of Management of St. Petersburg University. This conference is the second of the St. Petersburg master-plan conferences on Game Theory and Management, the first one of which took place also in this city just one year before.

This conference held in new millennium is not unique as the second international conference on Game Theory and Management since after the first conference GTM2007 another international workshop on Dynamic Games and Management was held in Montreal Canada. Because of the importance of the topic we hope that other international and national events dedicated to it will follow. Starting our activity in this direction last year we had in mind that St. Petersburg University was the first university in the former Soviet Union where game theory was included in the program as obligatory course and the first place in Russia where School of Management and Graduate School of Management was established.

The present volume contains abstracts accepted for the Second International Conference on Game Theory and Management, held in St. Petersburg, June 26-27, 2008. There are five invited lectures; their abstracts are also included. As editors of the Volume 2 of Contributions to Game Theory and Management we invite the participants to present their full papers for the publication in this Volume.

Acknowledgements. The Program and Organizing Committees thanks all people without whose help this conference would not have been possible: the invited speakers, the authors of papers all of the members of Program Committee for referring papers, the staffs of Graduate School of Management and Faculty of Applied Mathematics & Control Processes.

Special thanks to Russian Foundation for Basic Research for the essential financial support (project 08-01-06065-г).

We would like to thank Maria Dorokhina, Andrey Zyatchin, Maria Yurova and Elena Paramonova for their effective efforts in preparing the conference. We thank them all.

Nikolay A. Zenkevich, GTM2008 Organizing and Program Committees

Generalized ‘Lion & Man’ Game of R.Rado

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Keywords: *Avoid, Capture, Game, Strategy*

Abstract: *It is considered the generalized game of R. Rado ‘Lion & Man’ when ‘Man’ moves along a given line.*

It can be considered as a first example of dynamic games the problem about ‘Lion’ (briefly L) persuading ‘Man’ (M) when the latter was running along the circumference while L moved inside the circle. By elementary but refined arguments R.Rado [1, 2] proved that L was able to capture (M) if speeds of the both points were equal. Further interesting results on ‘Lion & Man’ game concerning the case when both points moved inside a circle, was obtained by J.O.Flynn [3, 4].

Here the generalized ‘Lion & Man’ game will be considered when M moves along a given line $\Gamma \subset \boldsymbol{R}^n$ with the speed σ while L can move all over the space $\boldsymbol{R}^n$ with the speed ρ . The line Γ is given by an absolutely continuous mapping $\gamma : \boldsymbol{R} \to \boldsymbol{R}^n$ and parameterized by arc length. Motions of the points are described by the equations $\dot{x} = u,\ \dot{y} = v$ with the phase constraint $y(t) \in \Gamma$ and control constraints $|u| \le \rho,\ |v| \le \sigma\ (x, y, u, v \in \boldsymbol{R}^n).$ The problem is interesting if $\sigma \ge \rho > 0$ and Γ is embedded into $\boldsymbol{R}^n$ isometrically to $\boldsymbol{R}$ or $\boldsymbol{S}$. (If Γ is isometric to half line $[0,\infty)$ then problem can be reduced to the first case.)

It can be supposed $x(0) = 0,\ y(0) = \gamma(0) \ne 0.$ Let $v = v_\nu + v_\tau$ be the decomposition such that v_ν is directed as y and v_τ is orthogonally to v_ν . The function

$$U_R(x, y, v) = \xi v_\tau + y(\rho^2 - \xi^2 \,|\, v_\tau \,|^2)^{1/2} \big/ |y|$$

will be called R -strategy [5], where $\xi = |x| / |y|$.

Theorem 1 [5, 6]. Suppose $\rho = \sigma$ and Γ is closed. Then R-strategy guaranties the capture i.e. there exists $T, T > 0,$ such that for any admissible control function $v(\cdot)$ of M $x(t) = y(t)$ at some $t \in [0, T]$ for the corresponding trajectories.

Note that the proof is not simple in contrast with the case when Γ is a circumference.

Now suppose Γ is not closed so $\Gamma \setminus \gamma(0)$ consists of two components Γ_+, Γ_-. Formulate the following Condition A:

There exists a point $\gamma(s_\pm) \in \Gamma_\pm$ that $|x(0) - \gamma(s_\pm)| = |s_\pm|$ correspondingly.

Obviously if Condition A did not occure for Γ_+ or Γ_- then M easily avoids a capture. It turns out the converse is also true.

Theorem 2 [6]. Suppose $\rho = \sigma$ and Γ is not closed and Condition A holds for both arcs $\Gamma_\pm$. Then L wins the game by using the R-strategy thereupon a manoeuver.

Further $\rho < \sigma.$

Theorem 3 [7]. Let the function γ' satisfies Lipschitz condition i.e.

$$|\gamma'(s_1) - \gamma'(s_2)| \le \lambda |s_1 - s_2|, \lambda > 0 .$$

Then there exist a positive constant $\mu = \mu(x(0), y(0), \rho, \sigma, \lambda)$ and a positional strategy of M such that for any admissible control of L the inequality $|x(t) - y(t)| \ge \mu$ holds for all $t, t \ge 0.$

Note Lipschitzianity of γ' is essential.

Now suppose that γ' has a jump discontinuity.

Theorem 4. Let Γ be a closed line having a corner point with an inside angle equal to $2\alpha,\ \alpha \in (0, \pi/2).$ If $\rho > \sigma \sin\alpha,$ then L wins the game.

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A Modified English Auction for an Informed Buyer

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Keywords: *Dynamic auction, Informed principal problem*

In procurement, when a company wants to buy a good or hire a service, it's rarely the case that just the price matters. On the contrary, several other factors are usually considered, such as quality, time of delivery, business relationship value,... Even though some of these elements enter in the range of matters that are the subject of bargaining between the buyer and the suppliers, there are others that are left outside. This is because some factors are non-contractible, and also because the buyer does not always fully reveal what really matters to her.

We derive the optimal auction in an environment where both buyer and suppliers hold some private information about their preferences, and we propose a dynamic mechanism that implements the optimal auction outcome in a very practical way.

We consider a setting where both the buyer and the suppliers have quasi linear utility functions and independent private values. Each supplier knows his cost of production, and the buyer knows how much each supplier's product fits her requirements. In other words, to each supplier corresponds a suitability value that is private information of the buyer. We analyze the problem as an Informed Principal Problem (Myerson (83)). We show that, if we rule out the possibility to use lotteries in the mechanisms, the optimal auction is the same as in an environment with common knowledge of the buyer's type.

Once derived the optimal auction outcome, we introduce three mechanisms that implement it: the Modified First Price Auction, the Modified Second Score Auction, and the Modified English Auction.

In the first format the allocation rule is such that for each supplier there is a scoring function that integrates the buyer's information with the supplier's bid. The winner is the supplier with the highest score. As in the standard First Price Auction, the winner is the only bidder who gets paid, and he receives a payment equal to his bid.

The second format is a variation of the Second Price Auction. The allocation rule uses a simple scoring function, the same for all suppliers, where the score is given by the difference between the suitability value and the bid. The winner is the supplier with the highest score. The payment rule is such that the winner is the only one who gets paid, and the amount he receives is a function of his suitability value and the score of the second best supplier.

The Modified English Auction is a dynamic mechanism. At the first round, the buyer makes an individual take-it-or-leave-it offer to each potential supplier. If a supplier refuses the offer, he drops out of the auction; if he accepts the offer, he stays active and passes to the following round. From the second round on, out of all the active suppliers, the buyer chooses one as the temporary winner, and makes a new offer to all the others. Whenever the buyer makes a new offer, he decreases the corresponding standing offer from the previous round by a small, discrete amount. Each offer is made directly to the supplier who is the intended recipient, and it is not disclosed to anyone else. When the temporary winner is the only active supplier left, the auction ends. The temporary winner becomes the auction winner, and he receives a payment equivalent to the last offer he accepted.

Considering the implementation problem, we focus on three aspects that are particularly critical in real business practice: privacy protection, dynamic consistency, and simplicity.

Companies that run procurement auctions are often very reluctant to fully disclose the criteria on the base of which the suppliers' offers are compared. This happens because these evaluations are based on technical and strategic considerations whose privacy is perceived by the management as critical for the company's future success. However, the choice of keeping secret many details of the mechanism may end up in compromising the functioning of the mechanism itself. For example, considering the three optimal mechanisms we introduced, if the buyer's private information is not disclosed, we show that the Modified First Price Auction fails to implement the optimal auction outcome, and that the Modified Second Score Auction becomes dynamic inconsistent. The strongest result of our work is to have designed a mechanism that is optimal, privacy preserving, dynamically consistent, and simple: the Modified English Auction.

We show that the flow of information the buyer receives in the Modified English Auction is such that she never deviates from the optimal strategy. On top of that, the confidential manner with which every offer is disclosed only to the intended recipient guarantees that any supplier never gets enough information to infer the buyer's type.

It's important to notice that, even considering standard settings, where the auctioneer does not hold any private information, the Modified English Auction is the first practical mechanism that implements the optimal auction in an environment with asymmetric bidders.

Hierarchical Games and Reverse Engineering

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This plenary lecture will introduce a class of hierarchical dynamic games with one leader and multiple followers, where the players are endowed with private as well as public information on some stochastic parameters that determine their individual preferences, and the leader has some partial information on the actions of the followers. With the followers taken as non-cooperative players, an appropriate solution concept for such games is the *Stackelberg* (or *leader-follower*) solution between the two levels of decision making and the non-cooperative *Nash* equilibrium among the followers. A direct approach towards derivation of the corresponding equilibrium policies involves infinite-dimensional optimization problems even if the plays' actions take values in finite-dimensional spaces, and as such is infeasible. An alternative indirect approach involves computation of the leader's best performance under a number of relevant informational constraints, and then *reverse-engineering* a policy for the leader under which given also the Nash responses of the followers the pre-computed performance level can be achieved — exactly or approximately.

The talk will first discuss the general elements of the *reverse-engineering* methodology, and then apply it to specific games that arise in the pricing of services in communication networks with a service provider (acting as the leader) and a population of users (being the followers). The users could vary in terms of their preference profiles, and these variations could be captured by some stochastic parameters, which are only partially available to other players (users as well as the service provider). The prices set by the service provider (SP) for the resources demanded by the users could be user dependent, to the extent the SP can differentiate between different types of users. After presenting policy constructions for such games and elaborating on their implications, the talk will conclude with a discussion of extensions to multiple leader scenarios.

Dynamic Games in Finance

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Keywords: *Dynamic games, Applications in finance, Numerical methods*

Various questions in the broad area of Finance that can be approached using the framework of dynamic games. In the area of Corporate Finance, the default decision, as well as the negociation process leading to liquidation or reorganization may be modeled as a non-cooperative or cooperative game between claimants (creditors and shareholders). In Investment Finance, the competition between mutual funds to attrack individual investors business can be represented by a non-cooperative strategic sequential game. Derivative securities that can be exercized by more than one decision maker, such as embedded options, are dynamic zero-sum games between the various holders. Financial services systems may be represented by competitive multi-agent models. This tutorial will present dynamic game models and propose numerical solution methods for such application examples.

Advertising and Product Price Policies in a Fashion Licensing Contract

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Keywords: *Advertising, Game theory, Licensing, Stackelberg*

Licensing an entity such a brand, a name or a logo requires a contractual agreement between two business subjects: the owner of the property, called licensor, and the renter of the rights, called licensee, see [3]. Both involved actors are interested in maximising their profits, which depend on the sales of a unique product, and their objectives can be achieved acting on communication and on price.

In [1], Buratto and Zaccour formulated a differential game, played à la Stackelberg, where the Licensor was the leader, and the licensee the follower, in order to plan the optimal communication policies. Cooperative and noncooperative advertising strategies of the two players were characterized analysing the Stackelberg equilibria in two particular types of fashion licensing contract: licensing in a same core business and licensing in a complementary business. The use of an incentive strategy by the licensor was considered too.

Here the licensing communication problem is studied again taking into account that price influences the demand and therefore considering not only the advertising expenditures, but also the retail price as a decision variable. The form of the demand function depending on the retail price is analogous to the one proposed by Jorgensen and Zaccour in [2].

Both types of fashion contract are analized and the Stackelberg equilibria are found explicitly.

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Strategy-proof Mechanisms of Organizational Control

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Keywords: *Active expertise mechanisms, Organizational systems control theory, Strategy-proofnes, Transfer of information*

Abstract: *We briefly survey a problem of strategy-proofness within the framework of the organizational systems control theory – a school that investigates mathematical models of organizational control mechanisms (decision-making routines). The basic model of an organizational system with uncertainty is formulated. Classic and current problems are listed.*

One of the key problems that arise while solving control problems in socio-economic systems is the problem of decision-making under uncertainty. The theory of organizational systems control distinguishes between two types of uncertainty (incomplete information) – internal and external. This division is based on the possibility to obtain the missing information from other members of organizational systems (so-called agents or active elements). The uncertainty that can be eliminated through the knowledge of other members of the system is internal, and the one that can not be eliminated – external. To effectively address the internal uncertainty the organizational control mechanisms with the transfer of information can be used. In such mechanisms the principal takes control decisions on the basis of information received from subordinate agents. The use of these mechanisms involves risk of opportunistic behavior of one or more agents, i.e. manipulation of the information reported (distortion of the information in their own interests and contrary to the principal's interests or interests of a system as a whole). A mechanism is said to be strategy-proof if it is dominant strategy incentive compatible, i.e. reporting the truth is a dominant strategy for each agent. The significance of strategy-proof mechanism design bases on the proved fact that for many problems of organizational control an effective mechanism can be found among strategy-proof ones.

The report presents an overview of results for the strategy-proof mechanism design obtained with the aid of the game-theoretic approach by Burkov's school of the theory of active systems (TAS).

A brief overview of classic results is offered: The fair play principle of control [1, 4, 5, 6], the method of dictatorship ranges [7], and a number of algorithms for strategy-proof mechanism design [1, 4, 5, 6]:

Resource allocation mechanisms

Active expertise mechanisms

Internal price mechanisms

Mechanisms of exchange

Mechanisms of agreement

These results are compared with results, obtained by Moulin [10], Border, Jordan [9], Barbera [8] etc.

In conclusion, one of the topical problems is stated – a problem of active expertise over multi-dimensional opinion spaces [2, 3]. A variety of strategy-proof mechanisms for this problem is offered, the problems of mechanisms feasibility and effectiveness are formulated and discussed.

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A Bargaining Approach to Negotiated Agreements Between Public Regulator and Firms

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Keywords: *Bargaining games, Nash bargaining equilibrium, Voluntary Approaches*

We develop a bargaining model between a public regulator and an industrial association to discuss the private contribution to a public good and the public financial support to the firms' effort.

We use a Nash bargaining equilibrium concept to establish the negotiation space and the equilibrium conditions for the agreement. The objective is to discuss the factors that influence that negotiation space and the structure of the equilibrium.

We make a mathematical deduction of an equilibrium expansion path and demonstrate the elliptic structure the of the iso-Nash Product contour curves. An economic intuition is given for both the equilibrium path and the elliptic configuration of the iso-Nash Product curves.

We conclude the theoretical development of the model with a comparison between the equilibrium conditions for a situation when the industrial association represents heterogeneous firms. A comparison is made between the results of a polling agreement (when the same agreed result is valid for all the firms) and a separated agreement allowing for different results for each type of firm.

The motivation for the analyses comes from the Portuguese experience with environmental agreements and, in particular, with the negotiated agreements that are a specific form of environmental voluntary agreements.

The negotiation between the public regulator and the firm comprises the environmental effort that the firms are willing to make and their predisposition to assume the financial and social costs related to the implementation and control of the environmental measure. The public regulator will grant financial support to the firms, expecting to lessen the social and political pressure.

The threat in case of negotiation failure is the inevitability of a public intervention through the implementation of a command and control measure.

In the case of public regulation, an environmental objective is forced to the firm and the implementation and control enforcement costs are borne by the regulator. The expected enforcement costs are directly dependent on the environmental audit costs and the fine for non compliance that is approved by a legislative body.

The payments for firms and regulator are given by cost functions that are influenced by the environmental effort, the financial support (a public subsidy) and enforcement cost. The negotiation is conditioned, not only, by the usual participation constrains, but also, by political constrains related both to the environmental objectives and the financial costs of the agreement for the regulator.

Our model enable us to demonstrate that the social costs of public funding and the technical efficiency of the firms influence the negotiation space as well as the equilibrium conditions. And we also demonstrate that the legislative body is able to influence de results of the negotiation through the decision on the fee for non compliance. This influence is enforced even though the legislative body does not participate in the negotiation, through its capacity to condition the position of the regulator in the negotiations.

The comparison between the polling agreement and a separated agreement results in the conclusion that the more flexible negotiation is well-fare enhancing.

The Shapley Value for Games with Restricted Cooperation

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Keywords: *Shapley value, Restricted cooperation*

Given a cooperative game (*N; v*), with characteristic function v and players set *N*, a restriction in the cooperation is given by a set system (*F;N*), where *F* is the set of feasible coalitions that can be formed. Many different restrictions have been considered; some examples are coalition structures (Aumann and Dreze, 1974), graphs (Myerson, 1977), partition systems (Kaneko and Wooders, 1982, Faigle, 1989), precedence constraints (Faigle and Kern, 1992), and some combinatorial structures as convex geometries, matroids, greedoids and antimatroids (see the survey of Bilbao, 2000). Our goal is to extend the Shapley value to any set system (*F;N*), where we only assume that the empty set belongs to *F*.

Lyapunov Stability Issues for Dynamic Coalition Games

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Keywords: *Driver associate, Coalition games, Adaptive control, Lyapunov methods, Stability analysis, Reachability*

This paper explores Lyapunov stability issues for dynamic coalition games with multi-agent formation and their objective functions. First, a stable set of trajectories between the desired and actual outputs of the adaptive controller is defined. The route guidance control parameters are modeled as the strategy space of agents. Then, the agents' strategies are based on Lyapunov stability theory in the presence of a class of state-dependent uncertainties so that they can be used to adaptively adjust the gains of the feedback controls. Subsequently, this will lead to a global stable-based set asymptotically. An important study insight is that the Lyapunov function for the route guidance control models is their corresponding objective function. The reachability property guarantees obstacle avoidance and computational tractability for real-time deployment. It assures that goal state is reachable and will not get stuck in a local stable set. Furthermore, this scheme has a faster convergence property and the stability is guaranteed. This general procedure of the stability analysis overcomes the key difficulty of constructing meaningful Lyapunov function interpretations for adaptive control system design.

The central goal of this paper is to explore a comprehensive methodology of dynamic coalition game for the design, testing, verification, and realization in the real-world deployment. This research involves a cross-disciplinary blend of ideas drawn from the differential game [1], adaptive control, and artificial intelligence as the base disciplines. Dynamic coalition game known as a cooperative agent-based system with the dynamic objective function has been studied for a long time. In the game, an agent will be referred to a rational decision maker that refers to when he has a number of choices. Each agent will evaluate his strategy space and choose the best one, based on his current situation. In artificial intelligence, it has been used the term agent, possibly with adjectives to yield an autonomous software component that can decide what he needs to do. Control theory usually refers to the decision maker as a controller or actuator. Decision making also comes in many flavors, depending on whether the agent has complete or partial knowledge of his world, whether he is acting alone or in collaboration with other agents. An agent should also learn to improve his performance over time, as he repeatedly performs the sense-think-act cycle in the game.

Increasingly, researchers have focused on deploying multiagent systems to support humans in critical activities, revolutionizing the way a variety of cooperative tasks. One promising application of interest to automobile industry is the Driver Associate project [2]. Driver Associate is based on the dynamic coalition game paradigm which consists of multiple agents. Agents are capable of performing a variety of tasks such as navigations, obstacles avoidance and communicating on behalf of human drivers. Unburdened by navigation or control tasks, humans would free up time for more productive decision making. To be able to perform its roles as an assistant, agents of Driver Associate will need to be endowed with a coalition [3].

We consider the question of how coalition game affects the overall quality of vehicle controls. The Driver Associate consists of a group of control agents who work autonomously to accomplish their assigned tasks or may team up with other agents doing coalition operations in an emergency occurrence. Agents are software components that control predefined parameters in their strategy space. Each agent's software runs on its own dedicated microprocessor. In a game-theoretic aspect, coalition agents cooperate to improving the safety driving against the nature environment. A player may not be an agent, unless he declares to join the cooperation on some stage of the game. At the extreme cases, all players can form a ground coalition and also a single player coalition will make other players as dummies. If a player is out of coalition, he would not form any anticoalition to harm any coordinate efforts. The payoff function is defined to be a system performance evaluation function. From the definition of payoff function, we may conclude that the goal of game is to approach a safety and stable final state with individual interactions among agents.

In multiagent systems, agents interact with each other and with their coalition manager agent to achieve coordinative behaviors. Coalition occurs in two conditions: (1) agents have a common goal and their actions tend to achieve the goal; (2) the agents perform actions that will not only achieve their goals, but also the goals of other agents. Coordination means that agents act in a way such that their community acts in a coherent manner. Coherence means that the agents' actions get well, and that they do not conflict with one another. Agents in this environment face with dynamic changing situations for his control resources. Agents taking different roles will face same goals and share the performance. The challenge in the situation is changing dynamically and rapidly, agents need to decide which strategies they should take based on the real-time feedback.

Dynamic coalition game is used to model of the world environment in which the agents act. To obtain the mathematical model of the agents' environment, a set of differential equations is usually specified. These equations are assumed to capture the main dynamical features of the environment. A characteristic property of this specification is that these dynamic equations mostly contain a set of so-called parameters. These parameters model the effect of the actions taken by the agents on the environment during the course of the game. The agents' strategies are formalized by the parameters as the coefficients of kinematics equation. The phase state variables in the reduced coordinates represent the positions (and perhaps velocities) of an optimal trajectory. Trajectory has to be guided for attaining a given terminal set. Lyapunov functions are used in objective functions to guarantee terminal sets to be stable and feasible for all time.

Stability is an important operational issue for the control of dynamic systems through adaptive controllers and route guidance. This is because inappropriate information dissemination or route guidance may lead to increased unpredictability and volatility, and/or catastrophic consequences for the dynamic system, compromising the objectives of game. Once the state space contains many obstacles, the optimal trajectory has to deviate from the previous adopted "optimal" trajectory to reach different possible intermediate stable state and connect to final goal state. The notion of stable reachability is to guarantee trajectory to reach final goal state. Regular reachability is of primary importance in optimal control problems with function space end condition, because in case of regularity Lagrange multipliers can be identified with functions. Regularly reachable sets characterize that suboptimal trajectory is a piecewise continuity [4].

An important contribution of the paper is that the Lyapunov functions $V(x)$, where x represents the phase state variable, are corresponding to agent performance functions. This procedure of the stability analysis is to declare the stable sets based upon performance functions related to obstacle region. It overcomes the key difficulty of constructing meaningful Lyapunov function interpretations for adaptive control system design [5]. Another advantage of the proposed approach is its ability to determine less conservative estimates of the region of attraction. Lyapunov function has a quadratic form which implies a region of attractions. Local stability analysis can imply a small region of attraction [6]. In our study, the estimated region of attraction is a relevant compact invariant set, which is a less conservative estimation of the attraction region. Hence, the proposed procedure is less restrictive.

In the context of real-time operations, the dynamical system search for the assignment strategy that leads to the stable desirable state (for the next assignment time interval) can be constructed at any point in time. This is because the analysis procedure always moves the trajectory toward to desirable states for passing over obstacles and implying an intermediate solution that is available along the search trajectory. The procedure is less computationally intensive than procedures that seek all equilibrium states. These ensuing trade offs between computational time and solution effectiveness provide a convenient handle for obstacle avoidance implementation.

The remainder of this paper is organized as follows: Section II presents the necessary notation and terminology and formulates the dynamic coalition game. Section III provides a rigorous stability analysis for describing the adaptive controller design procedure. Section IV discusses the relationship between reachability property, obstacle avoidance and computational tractability for real-time deployment. The paper ends with the concluding remarks and future works in Section V.

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Informational Structure Transformation in Reflexive Games

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Keywords: *Informational structure, Reflexive decision-making, Reflexive game*

Abstract: *The paper contains the game-theoretical model of reflexive decision-making. If the normal form reflexive game is repeated several times, some agents (or even all agents) observe the result (choices of opponents, values of goal functions etc.), different from the expected one. It may be caused both by incorrect beliefs about the state of nature and by inadequate information about opponents' beliefs. In this case informational structure of the game is changed.*
The paper is devoted to the consideration of three problems: informational structure, actions of the agents based on it, informational structure transformation.

Game theory studies models of decision making by several participating decision makers. It is usually assumed that the decision makers (thereafter called agents) are rational. However, the information available to agents could differ and this could significantly impact their decisions.

This presentation is a continuation of a series of works (see for ex. [2-4]), modeling decision making in presence of incomplete information, using the method of reflexive games (alternative is models based on Bayesian games, most completely surveyed in [1]).

1. Informational structure

Let us describe information structure available to agents in presence of incomplete information.

We consider n players, called *real agents*. Introduce the following notions and sets (all sets are assumed finite).

Θ – the set of states of nature (or simply states);

A_i – the number of copies of agent i, $i \in N = \{1, \ldots, n\}$;

$A = A_1 \cup \ldots \cup A_n$ – the set of all agents;

$\Omega \subset \Theta \times A_1 \times \ldots \times A_n$ – the set of all *possible worlds*.

In each possible world $\omega = (\omega_0, \omega_1, \ldots, \omega_n)$ we have a state $\omega_0 \in \Theta$ and a copy $\omega_i \in A_i$ of an agent. We will say that agent ω_i belongs to world ω.

η – the *information function* of an agent, which to every agent $a \in A$ assigns a set of worlds $\eta(a) \subset \Omega$, which the agent assumes possible due to the information available to him.

$\omega^* \in \Omega$ – *real world.* One of the possible world is assumed to be real, i.e. it is characterized by the state ω_0^* and agents ω_i^*, which actually exist.

The agents belonging to the real world are real, the remaining agents are called phantom agents [1-3].

We assume that the following conditions hold.

Condition 1 (identity of the agent).

$\forall i \in N$, $\forall a_i \in A_i$, $\forall \omega \in \eta(a_i)$ we have $\omega_i = a_i$. Namely, each agent belongs to all worlds which he assumes possible.

Next for each world ω we define a set of worlds $I(\omega)$, *connected to the world* ω, as follows.

World ω' is connected to world ω^1 if there exist a finite sequence of worlds $\omega^2, \ldots, \omega^m$ and agents $a_{i_1}, \ldots, a_{i_m}$ so that

$$a_{i_1} = \omega_{i_k}^k, \; k = 1, \ldots, m,$$

$$\omega^{k+1} \in \eta(a_{i_k}), \; k = 1, \ldots, m-1,$$

$$\omega' \in \eta(a_{i_m}).$$

Agent is connected to world ω' if he belongs to the world connected to ω'.

The notion of worlds and agents connected to a given world allows for introduction of the second condition.

Condition 2 (unity of the world).

Each world and each agent is connected to the real world:

$\omega \in I(\omega^*)$, $a \in I(\omega^*)$, for every world $\omega \in \Omega$ and every agent $a \in A$.

The tuple $(\Theta, A_1, \ldots, A_n, \Omega, \omega^*, \eta(\cdot))$, where

$\Omega \subset \Theta \times A_1 \times \ldots \times A_n$, $\omega^* \in \Omega$,

η: $A_1 \times \ldots \times A_n \to \exp(\Omega)$

and Conditions 1, 2 are satisfied, will be called *(multiple) informational structure*.

The informational structure will be called *correct* if for every agent there exists a world which he assumes possible:

$\forall\ a \in A\ \eta(a) \neq \varnothing$.

2. Informational equilibrium

Let $\theta \in \Theta$ be a state of the world, and let $x_i \in X_i$, be an action chosen by the i-th agent. The actions are chosen by agents simultaneously and independently, i.e. we consider a game in a normal form.

Furthermore, let $f_i(\theta, x_1, ..., x_n)$, $i \in N$ be the goal function of the agent, and assume that the informational structure is correct. Then a collection of functions χ_i: $A_i \to X_i$, $i \in N$ is called *informational equilibrium* if

$$\chi_i(a_i) \in \operatorname{Arg} \max_{x \in X_i} \min_{\omega \in \eta(a_i)} f_i(\omega_0, \chi_1(\omega_1), ..., \chi_{i-1}(\omega_{i-1}), x, \chi_{i+1}(\omega_{i+1}) ..., \chi_n(\omega_n)).$$

In other words, each agent maximizes worst case result he assumed possible.

3. Transformation of informational structure

The informational structure represents sort of a ”snapshot” of the mutual information available to the agents. It is clear that the available information can change. In particular, information can change due to observations of the results of the game, and this results in transformation of the informational structure.

Recall that we consider a game in a normal form, namely the actions are chosen independently and simultaneously. As a result of the game, the information available to agents can change and the next game (if it takes place) they will play using new information.

Suppose the i-th agent possesses an *observation function* $w_i = w_i(\theta, x_1, ..., x_n)$ which is the common knowledge (for observation function see [3]). The meaning of this function is as follows: if the state of nature is θ and agents chose actions $(x_1, ..., x_n)$, then i-th agent observes value $w_i \in W_i$, where W_i is the space of all possible observations by the i-th agent.

The transformation of the informational structure means the following: for each agent (real and phantom) the set of worlds he assumes possible is being modified. The modification consists of excluding the worlds for which the value of the observation function is different from the one observed by the agent.

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Continuously Stable Strategies and Dominance Criteria for Evolutionary Games

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Keywords: *CSS and NIS, Dominance, Evolutionary games*

Abstract: *The Continuously Stable Strategy (CSS) and Neighborhood Invader Strategy (NIS) concepts, originally developed as intuitive static conditions to predict the dynamic stability of a monomorphic population, are shown to be closely related to classical game-theoretic dominance criteria when applied to continuous strategy spaces. Specifically, for symmetric and non symmetric two-player games, a CSS in the interior of the continuous strategy space is equivalent to neighborhood half-superiority while an NIS is equivalent to full neighborhood superiority. These conditions generalize risk dominance and* p^* *- dominance concepts for two-strategy two-player games.*
The CSS and NIS are also important for dynamic stability under the replicator and best response dynamics as well as for adaptive dynamics. In particular, these dominance criteria applied to two-species models give a game-theoretic method to predict the equilibrium behaviour of interacting populations.

Cressman (2007) considered the three concepts given in the following definition for a symmetric game with pure strategy set S and payoff function $\pi : S \times S \to \mathbf{R}$.

Definition 1 *Suppose* S *is a subinterval of* $\mathbf{R}$ *and* $x^* \in S$.

(a) x^* is a *neighborhood CSS* if the following two conditions hold.

(i) There exists an $\varepsilon > 0$ such that, for all $x \in S$ with $0 < |x - x^*| < \varepsilon$, $\pi(x^*, x^*) > \pi(x, x^*)$ (neighborhood strict Nash equilibrium condition).

(ii) There exists $\eta > 0$ such that, for all $x' \in S$ with $0 < |x' - x| < \eta$, $\pi(x', x) > \pi(x, x)$ if and only if $|x' - x^*| < |x - x^*|$ (convergence condition).

(b) x^* is *neighborhood* p^* *– dominant for two-strategy games* if there exists an $\varepsilon > 0$ such that, for all $x \in S$ with $0 < |x - x^*| < \varepsilon$, $\pi(x^*, q) > \pi(q, q)$ for all mixed strategies $q = px^* + (1-p)x$ in the symmetric game with strategy set $\{x^*, x\}$ for which $p^* \leq p \leq 1$.

(c) x^* is *neighborhood* p^* *– superior* if there exists an $\varepsilon > 0$ such that $\pi(x^*, P) > \pi(P, P)$ for all mixed strategies P with strategy set $\{x \in S : |x - x^*| \leq \varepsilon\}$ for which $p^* \leq P(\{x^*\}) \leq 1$.

If x^* is a neighborhood CSS, then x^* is a CSS according to the definition introduced by Eshel (1983) when the strategy space is restricted to those $x \in S$ sufficiently close to x^*. Similarly, for this restricted strategy space, x^* is neighborhood p^* – dominant if and only if it is p^* – dominant in all two-strategy games according to the definition introduced by Morris et al. (1995). Note that, in part (c), P is a probability distribution over the continuous strategy space with weight at least p^* on the point x^*.

Cressman (2007) (see also Cressman, 2004) shows the three concepts in the above definition are equivalent when $p^* = \frac{1}{2}$ and the payoff function has continuous non-zero second-order partial derivatives. The talk will show generalizations of this result to symmetric games whose continuous strategy space S is a convex subset of $\mathbf{R}^n$ and to two-player non-symmetric games with continuous strategy spaces. The extension of the CSS concept in part (a) to these cases is based on its interpretation as an asymptotically stable monomorphism under the canonical equation of adaptive dynamics that models the effects of small but arbitrary mutations.

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A Two Level Game-theoretical Hierarchical Model of Plural Corruptive Interaction

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Keywords: *Corruption, Hierarchical control*

Abstract: *Two-level hierarchical mathematical model of resource's quota distribution including plural corruption is described. The Follower's reaction domains are considered. The description of the applicability's limitation of the impulsion method is provided.*

In the article the attempt to aggregate the set of distinct two-level game-theoretical models [1] of two person corruptive interaction into a single game-theoretical was made. Also, the ability to provide few bribes of the different nature to perform different illegal actions by the Leader's side is touched upon. The actions abovementioned might be interconnected and profitable (in general case) for both the Leader and the Follower. Also, one of the aims of the research was the attempt to reveal and estimate the consequences of the compulsory Leader's corruptive efficiency reduction if the hierarchical control methods are applied by the Leader's side. Moreover, the complete implementation of interactions connected with the unprincipled Leader's behavior to obey the liability to follow the corruptive agreement with the Follower could be regarded as one of the primary model's enhancements. Compulsion and impulsion methods are applied as the primary hierarchical control methods in the model.

As the result of the research performed and the enhancements the following model was formulated:

The Leader's payoff function is the following:

$$J_{v} = (c_1 * p_\beta * u^{k_1} - M * \rho(U,u)) \to \max_{q \in Q_v} \max_{p \in P^{U(q)}} \min_{u \in R(p,q)}; \tag{1}$$

The Follower's payoff function is the following:

$$J_{u} = (c_2 * (1 - p_\beta) * u^{k_2} * (1 - \beta_p - \beta_q) - M * \rho(U,u)) \to \max_{u, \beta_p, \beta_q}; \tag{2}$$

The contingencies are shown below:

$$\begin{cases} q_\beta = q - \sigma * \beta_q + d_q; \\ u \in [0, 1 - q_\beta]; \\ U = [0, a]; \\ p, q, p_\beta, q_\beta \in [0,1]; \\ p_\beta = p - \gamma * \beta_p + d_p; \\ d_p \in [0, \gamma]; \\ d_q \in [0, \sigma]; \\ a \in [0,1]; \\ k_1, k_2, \gamma, \sigma > 0; \\ \beta_p, \beta_q \geq 0; \\ M -> +\infty; \end{cases}$$

The model's parameters described in the following way:

U – sustainable development domain; a – sustainable development domain magnitude; u – resource extraction contents (the Follower's control parameter); p – tax ratio (the Leader's control parameter); $1-q$ – quota to the resource extraction (the Leader's control parameter); M – penalty constant; $\rho()$ – penalty function; γ – p-corruption's efficiency (the price of illegal tax ratio variation); σ – q-corruption's efficiency (the price of illegal quota variation); d_p – p-corruption rigidity (the ratio that reflects the value of the Leader's unability to follow the corruptive agreement burden in respect of taxation policy); d_q – q-corruption rigidity (the ratio that reflects the value of the Leader's unability to follow the corruptive agreement burden in respect of quota distribution); β_p and β_q parameters reflect according bribe values and they are treated as the Follower's control parameters.

The players behavior submits to Germeyer's approach (the information is decided to be complete):

The Leader estimates the Follower's optimal reaction domains that are subject to the hierarchical control methods application. Also, the Leader calculates his expected final income, assuming that the Follower's reaction is the most unprofitable for him. After that, the Leader chooses his strategy based upon the abovementioned estimations. Then the Follower reacts in optimal way for him that makes the situation being formed.

The Follower's optimal strategies were found for the model formulated. Also, five different reaction domains were separated regarding the optimal strategy classes.

The Leader's optimal strategies were found for all the Follower's reaction sub-domains. However, it is found that some sub-domains provide the equal income for the Leader that result in the unclear way for him to choose his optimal strategy (the stochastic approach is decided to be unsuitable for the current model's version). Hence, to avoid the uncertainty in such cases the factor of (un) favour to the Follower was implemented into the model. Moreover the applicability of the hierarchical control methods used was tested. There were found no additional limitations to the compulsion method in the models analyzed. Though, a class of heavy structural limitations for the impulsion method applicability was discovered, which could be treated as corollary fact from the model parameters interconnection. It could be described as the following: If the certain (rather soft) conditions are met, the impulsion's mechanisms classical conception becomes inapplicable. The problem lies in the following vicious circle: $u^* = u^*(\beta_q^*) = u^*(\beta_q^*(p)) = u^*(p)$. Hence, the Leader has no ability to specify the penalty-stimulation system in respect to the taxation policy because he becomes unable to determine the set of the extraction contents that is optimal for the Follower. But the set depends on the tax value that should be specified by the Leader himself. Such a class of limitations could be treated as unavoidable for the impulsion method's classical approach.

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Stability of Behavior in Prisoner Dilemma: Approach by Evolutionary Games

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Keywords: *Prisoner dilemma, Cooperation, Evolutionary games*

Abstract: *In this paper, we study the behavior in prisoner dilemma. We illustrate the influence of the degree of cooperation and the length of game on the emergence of the cooperation in an evolutionary setting.*

Evolutionary game theory is a branch of game theory which developed by Maynard Smith and Price (1973). In this theory each individual chooses among alternative actions (or behaviors) whose payoff or fitness depends on the choices of others. The basic idea is that actions (or behaviors) which are more "fit", given the current distribution of behaviors, tend over time to displace less fit behaviors (Friedman, 1991). The behavior of a player is compared to his degree of cooperation which reflects his capacity to support the cost of cooperating before punishing the deviant from the cooperation. If this degree is high, it will be able to encourage the competitor player to defect. And if this degree is low, it protects it from opportunistic behavior. The classical game theory is silent as to the manner to arrive at Nash equilibrium. The evolutionary framework provides a precise answer to this question since it considers the dynamics of the system. This dynamic process makes it possible to place the analysis within an evolutionary framework which allows the use of the refined concept of the Nash equilibrium: Evolutionarily Stable Strategy (*ESS*).

Von Neumann and Morgenstern (1944) pointed out that the players are assumed to be rational and selfish, and the utility scale is that of financial gain. Axelrod and Hamilton (1981) and Axelrod (1984) demonstrated that the reciprocity plays a principal role in emergences of cooperation. Axelrod (1984) showed that altruistic behavior is commonly attributed to inclusive fitness, a population containing substantial fraction of altruistism can be invaded by defectors. Wahl and Nowak (1999) are found that the initial degree of cooperation play the important role for the evolutionary robustness in the case of continuous prisoner's dilemma.

In our case of discret prisoner's dilemma, we study the reciprocal, selfishness and altruism behaviors within the framework of an interaction structured by prisoner dilemma. The altruism behavior is stable neutrally but the emergence of cooperation is closely dependent with reciprocal behavior. By introducing the degree of cooperation and the length of game, we demonstrated that if the proportion of individuals having the reciprocal behavior is greater to a certain limit, then the cooperation emerges. Thus, this study allows us to answer the question with which Axelrod begins his book “evolution of cooperation (1984)”: how we can explain the emergence of cooperation in a selfish population without intervention of the central power?

Multistage Biddings with Risky Assets: the Case of N Participants[1]

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Keywords: *N-person repeated games, Incomplete information, Multistage biddings, Random walk*

We develop and investigate a model of multistage biddings for risky assets (shares) with several participants possessing different information on the real value of shares. Unlike the model investigated in [1-4] with two players only and direct biddings between them, the new model include an arbitrary number of players and an auctioneer who conducts the biddings. This model seems to be more realistic.

A joint-stock company organizes multistage biddings to distribute a new lot of shares among company stockholders. The realization price of one share depends on random events which took place before the start of biddings, e.g., on transacted contracts, outcomes of research programs and so on. Player 1 has the "insider" information and knows the real value of share. All other participants know that Player 1 is an insider. The biddings model is organized in the following way:

1) Before the biddings start a chance move determines the non-negative integer realization price of a share for the whole period of biddings according to the probability distribution p. Player 1 is informed on the outcome of the chance move, the other participants are not. All players know the probabilities of chance move.

2) At each step of the biddings all players simultaneously declare their bids. Any non-negative integer bids are admissible.

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3) Each player, who declared the maximal bid, buys one share for this price from the auctioneer. Transactions occur only if the bids do not coincide completely.

4) After each step the debit of the company, i.e. the fetched money minus the expected price of sold shares, is divided in equal parts between all players (stockholders).

Each player aims to maximize the total increment of his portfolio, i.e. money plus realization price of acquired shares.

In this model the uninformed players should use the observed moves of the insider to reestimate their prior information and to update their believs on the real value of a share. Player 1 should use his private information without revealing it to other players. This enforces him to randomize his actions.

The described model of n-stage biddings is reduced to an N-person repeated game with lack of information at all players except Player 1. Analyzing these games we essentially base on our previous results for the games modelling two-person biddings without mediator.

We prove that if the price of share determined by distribution p has a finite dispersion $D[p]$, then the sequence of Player 1' guaranteed payoffs in n-stage games is bounded from above and converges. The limit is equal to a continuous concave piecewise linear function $H_N(p)$. Its domains of linearity are the sets of distributions p with the expected price of a share $E[p]$ belonging to the interval $[r, r+1]$, where r is an integer number. If $E[p] = r + x$, where $x \in [0,1]$, then $H_N(p) = (D[p] - x(1-x))(N-1)/2N$.

This result allows us to define correctly the games of unlimited beforehand duration. We show that this game has a unique subgame perfect equilibrium. At this equilibrium the insider payoff is equal to his guaranteed payoff $H_N(p)$ and the payoff of any other player is equal to his guaranteed payoff $-H_N(p)/(N-1)$. Hence, it is a strong subgame perfect equilibrium: any deviation of all players, but one, does not lead to payoff diminution of steady Player.

For distributions p with integer expectations $E[p] = r$, Player 1's equilibrium strategies are given with the following rules:

a) If the state $s = r$, then his total potential gain is equal to zero, and he stops the game. Thus the probability of stopping is p_r.

b) At state $s \neq r$ the first move make use of two bids $r-1$ and r. These bids are used with equal total probabilities $(1-p_r)/2$ and have the posterior probability distributions $p(r-1)$ and $p(r)$ with the expectations $E[p(r-1)] = r-1$ and $E[p(r)] = r+1$. The equalities $p_r(r-1) = p_r(r) = 0$ hold.

c) Further he plays in accordance with the posterior distributions $p(r-1)$ and $p(r)$.

These rules completely define the infinite Player 1's strategies for p with $E[p] = r$, $r = 1,2,\ldots$. These strategies generate symmetric random walks of expected share prices over the set of integer numbers. The walk stops at the moment when the expected share price turns to be equal to its real price. This is Markov time of revealing the real share price by uniformed players and the biddings termination.

The expected duration of this random walk is equal to the dispersion $D[p]$. Thus, if the share price has a finite dispersion $D[p]$, then the biddings end after a finite expected number of steps. The best answer of other players to the Player 1 equilibrium strategy guarantees Player 1 the one-step gain of $(N-1)/2N$. This makes clear the result $H_N(p) = D[p](N-1)/2N$, i.e. Player 1's total gain is equal to his fixed one-step gain multiplied by the expected number of steps.

We write out explicitly the random sequence of transaction prices, generated by the equilibrium strategies of players for infinite game. We show that at each step: a) probability that transaction occurs, i.e. the bids do not coincide completely, is equal to 1/2; b) under condition that transaction occurs, the expected transaction price is equal to the expected share price, i.e. the company debit is zero; c) under condition that transactions occur, the random sequence of transaction prices reproduces the random walk of expected share prices.

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On the Computation of Semivalues for Cooperative Transferable Utilities Games

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Keywords: *Shapley value, Cooperative games*

For a cooperative transferable utilities game, derived from an allocation problem, a traveling salesman problem, or some other models (see [1]), the most famous solution is the Shapley value, given by the well known Shapley value formula (see [10]). More recently, the Semivalues (see [8]) are also solutions, as a class of weighted values containing the Shapley value. In an earlier paper (see [7]), we have shown that any Semivalue, corresponding to a sequence of weight vectors, connected by a normalization condition and inverse Pascal triangle conditions, is the Shapley value of a game obtained by rescaling from the given game. This result was based on the relationships between any Semivalue and a Least Square value (see [5]), as well as any Least Square value and a Shapley value (see [6]), replaced in [7] by a direct relationship between Semivalues and the Shapley value. Therefore, an increased interest appears around the problem of computing the Shapley value by an algorithm which may be extended to the computation of the more general Semivalues.

In this paper, we return to an idea used by M.Maschler for developing an algorithm for computing the Shapley value (see [9]). Based upon the potential basis for the space of games with a fixed set of players introduced by the author in [2], we derive an accelerated algorithm for computing the Shapley value in exactly n steps. This will be done in the second section, where an example for a 4-person game is shown. Further, the relationship between the Semivalues and the Shapley value, proved via the Average per capita formulas (see [3], [4]), allowing the computation of a Semivalue as a Shapley value is explained in the third section. How this relationship can be used in the computation of Semivalues, together with an example illustrating the algorithm, are given in the last section.

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Deterministic Minimax Impulse Control

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Keywords: *Differential games, Impulse control, Quasi-variational inequalities, Viscosity solutions*

Abstract: *We consider a two-player differential game where the first player acts with a continuous control and the second one uses jumps. This minimax impulse control problem is motivated by an application to Finance. The dynamic programming principle leads us to show that the value function of the problem is a viscosity solution of a related Isaacs quasi- variational inequality (QVI). Under some natural assumptions on the dynamics and the costs, the uniqueness for this QVI is proved by assuming that the impulse cost is positive, using a fixed-point argument.*

1 The problem

1.1 System

1.1.1 Dynamics

Let a two-player differential game system be defined by the solution of following dynamical equations

$$\begin{cases} \dot{y}(t) &=& f(t,y(t),\tau(t)), \\ y(t_0) &=& x \in \mathbb{R}^n, \\ y(t_k^+) &=& y(t_k^-) + g(t_k, y(t_k^-), \xi_k),\ t_k \geq t_0,\ \xi_k \neq 0. \end{cases}$$

Here and below, the time variable belongs to $[t_0,T]$ where $T > t_0 \geq 0$ are given. The state at time t, $y(t)$ lies in $\mathbb{R}^n$.

The system is driven by two controls, a "continuous" control $\tau(t) \in \mathbf{K} \subset \mathbb{R}^\ell$, where $\mathbf{K}$ is a compact set, and an impulsive control defined by a finite sequence of impulse times t_k and the controls $\xi_k \in \mathbb{R}^m$ controlling the jumps in $y(t_k)$. Let $\psi = (\{t_k\},\{\xi_k\})$, where $k \in \mathbb{N}$.

For any initial condition (t_0,x) and controls $\tau(\cdot)$ and ψ generating a trajectory $y(\cdot)$ of this system, let a pay-off J be defined as

$$J(t_0,x,\psi,\tau(.)) = \int_{t_0}^{T} L(t,y(t),\tau(t))dt + \sum_k C(\xi_k) + G(y(T)).$$

We define the value function of the problem $W:[0,T]\to \mathbb{R}$ as

$$W(t_0,x) = \inf_{\Phi\in\Pi} \sup_{\tau(.)\in\Omega} J(t_0,x,\Phi(\tau(\cdot)),\tau(.)) \ . \tag{1}$$

For any bounded function $V:[t_0,T]\to \mathbb{R}$, we introduce also the operator $\mathbf{M}$ given by

$$\mathbf{M}\ V(t,x) = \inf_{\xi}\left[V(t,x+g(t,x,\xi)) + C(\xi)\right]$$

In the domain $[t_0,T[\times \mathbb{R}^n$, we consider the QVI

$$\max\left\{\min_{\tau\in\mathbf{K}}\left[-\frac{\partial W}{\partial t} - \frac{\partial W}{\partial x} f(t,x,\tau) - L(t,x,\tau)\right], W(t,x) - \mathbf{M}\ W(t,x)\right\} = 0 \tag{2}$$

with the terminal condition: $W(T,x) = G(x)$ in $\mathbb{R}^n$.

1.1.2 Regularity assumptions

In all the paper, we assume the following

1. $f(t,y,\tau)$ is Lipshitz continuous in y uniformly in t and τ with constant c_f , and it is uniformly continuous in t uniformly in y and τ .
2. The function $g(t,y,\xi)$ is Lipschitz continuous with respect to t , uniformly in y and ξ , with constant c_{gg} and it is Lipschitz continuous with respect to y uniformly in t and ξ , with constant c_g .
3. L , f , and G are bounded.
4. $L(t,x,\tau)$ is Lipschitz continuous in x uniformly in t and τ , with constant c_L , and is uniformly continuous in t uniformly in x and τ .
5. The function C is supposed such that $\inf_{\xi} C(\xi) = \gamma > 0$.
6. G is Lipschitz continuous with constant c_G .

2 Main results

Theorem 2.1 *Under the assumptions 1, 2, 3, 4, 5 and 6, the value function* W *is bounded, Lipschitz continuous in* t *, uniformly in* x *and it is Lipschitz continuous in* x *uniformly in* t *.*

Theorem 2.2 *The function:* $(t,x) \mapsto W(t,x)$ *is a viscosity solution of the quasi-variational inequality (2).*

3 Conclusion

We give the result of this paper.

Theorem 3.1 *Under Assumptions in Paragraph 1.1.2, the quasi-variational inequality (2) has a unique bounded uniformly continuous viscosity solution W.*

As an example of a use of this resut, one may consider the option pricing problem of references [5, 6]. If the piecewise linear transaction costs are replaced by a more realistic piecewise affine cost, i.e. a fixed cost is charged for any transaction in addition to a variable part, then the problem at hand is exactly that considered here. This was actually the motivation for the present analysis. The problem with no fixed cost, investigated by other means in these references, leads to a more difficult problem in terms of uniqueness of the viscosity solution, since it corresponds to the case $\gamma = 0$ in this paper. As far as we know, the uniqueness of the bounded uniformly continuous viscosity solution in that case is still an open problem.

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Some Variations on the Barro-Gordon Game: Why Null-inflations is the Better Equilibrium

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Keywords: *Dynamic game theory, Monetary policy, Stochastic differential games*

In this article we study game theoretic models of the conflict which arises between a monetary authority and the private sector with regard to the inflation-rate. Building up on the simple but illustrative one period game theoretic model introduced by Barro and Gordon [2], in our first model we assume that rather than playing a one shot game, monetary authority and private sector react to each other repeatedly for an infinite number of times. Both, the monetary authorities's and the private sector's reactions are assumed to be stochastic in the form of fixed behavioral transition probabilities. These probabilities are interpreted as strategies in a new game. We study the set of Nash-equilibira of this new game and how these correspond to the classical discretionary Nash-equilibrium of Barro-Gordon as well as the classical Non-Nash low inflationary state. In contrast to Barro-Gordon we show that the low-inflationary state can be realized as a Nash-equilibrium in our model. In a second model we investigate the same problem in a framwork of stochastic differential game theory. After introducing a general model, we focus on the case where the private sector has only partial information. The results presented in this talk are taken from the recent article "STOCHASTIC REACTION STRATEGIES, THE BARRO-GORDON FRAMEWORK AND HOW NULL-INFLATION CAN BECOME AN EQUILIBRIUM" available online at http://papers.ssrn.com/sol3/papers.cfm?abstract_id=1012566 as well as a current working paper.

Mutual Choice Problem[1]

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Keywords: *Mutual choice, Dynamic game, Equilibrium*

Abstract: *Mutual choice problem is considered. In the problem the individuals from two groups want to form a long-term relationship with a member of the other group, i.e. to create a couple. We present the dynamic game with n periods in which free individuals from different groups randomly meet with each other in each period. If free individuals accept each other, they leave the game. In the last period n the individuals who don't create the couple receive zero.*
In this paper we present the different models of mutual choice problem. In the first model the players create the couple with a highly ranked individual of other group, and in the each following period the new individuals arrive into the game. If free individuals accept each other, they leave the game and each receives the other's quality as a payoff.
In the second model the player who creates the couple receives the arithmetic mean of the qualities of the couple. Furthermore in the next period each player gets additional profit c.
The optimal strategies for players are obtained and numerical results are given for presented models.

1. Mutual choice problem with arrived individuals

In this section we consider the mutual choice game with arrived individuals. In the game the individuals from two groups (females and males or employers and workers, etc.) want to form a long-term relationship with a member of the other group, i.e. to create a couple. We present the dynamic game with n periods in which free individuals from different groups randomly meet with each other in each period. If they accept each other, they create a couple and leave the game. Let (x, y) are the qualities of the members from respective group (for example, level of fitness for mating problem, level of income for job search problem). The initial distributions of qualities are both uniform on [0, 1]. In the game the players create the couple with a highly ranked individual of

[1] The work is supported by Russian Found for Basic Research, project 06-01-00128-a and 08-01-98801-r-sever-a.

other group. If free individuals accept each other in the i - th period ($i = 1,2,...,n-1$), they leave the game and by symmetry each receives payoff u_i, but in the each following period the new individuals arrive into the game. We assume that there is a costant stream of the new individuals. In the last period n the individuals who don't create the couple receive zero.

Consider the case $n = 2$. In the first period the couple is created if $x \geq w$ and $y \geq w$, where w is the mean quality of the player in the next period. In the second period the new individuals arrive into the game. We assume that stream of the new individuals is equal to Δ.

The optimal w for the player is obtained from the equation

$$w = \frac{1+\Delta}{A_2}\int_0^w x dx + \frac{w+\Delta}{A_2}\int_w^1 x dx,$$

where $A_2 = w + (1-w)w + w\Delta + (1-w)\Delta.$

2. Mutual choice problem with additional profit

In the second model the player who creates the couple in the first period receives the arithmetic mean of the qualities of the couple. Hence the rule of acceptance is to create the couple if $\frac{x+y}{2} \geq w$, where w is the expected payoff in the second period. Furthermore in the next period each player gets additional profit c. We obtain the optimal value w and the optimal strategies for players depending on the profit c.

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Lottery Voting: May Majorities Prefer to Take a Chance?

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Keywords: *Lottery, Majority, Voting*

This paper analyzes and compares two voting procedures in elections between two alternatives under complete information: Majority voting and Lottery voting. When voting is costly, supporters of the majoritarian alternative suffer a collective action problem. This free-riding problem translates into a low turnout rate (that tends to zero as the size of the electorate increases) and on relatively low probability of winning the election, referred to as the "underdog effect". On the other hand, lottery voting selects a winning alternative with a probability proportional to the number of votes it obtains in the election. Although apparently surprising, this procedure was employed in ancient Athens and in the Italian Republics for centuries. We explore under which circumstances, lottery voting can successfully alleviate free-riding and investigate the cases in which it can yield a higher probability of success for the most supported alternative than majority rule. We show that when one group of supporters is much more numerous than the other and the cost of voting is sufficiently high, lottery voting considerably softens the "underdog effect". Moreover, it yields a higher expected level of turnout than majority rule. We then explore a possible caveat lottery voting, namely the victory of minoritarian and "dangerous" alternatives by introducing asymmetry of preferences within society.

A New Method to Check whether a Simple Game is Weighted[1]

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Keywords: *Trade robustness, Voting systems, Weighted games*

In binary voting systems and game theory in general there is an important problem that consists in determining whether a simple game can be represented as a weighted game, i.e. by means of a quota and non-negative weights associated to players (or voters). Thus, in a weighted game a coalition is declared to be winning if and only if the sum of the weights of the members that form it equals or surpasses some preset quota.

There exist some different results that deal with the problem. Perhaps the most natural way to cope with it is by solving an efficient system of inequalities [1]. However, the most well-known method involves trades among players in lists of coalitions. Indeed, a simple game is weighted if and only if it is trade robust [3, 6]; understanding that a simple game is trade robust if and only if it is k-trade robust for all positive integer k, and that a simple game is k-trade robust if a sequence of k or fewer (not necessarily distinct) winning coalitions can never be rendered losing by interchanging k players.

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The goal of this work is to give a refinement on the trade robustness notion and to illustrate the advantages of applying this new method to check whether a simple game is weighted.

To this purpose we consider what we call "invariant-trade robustness" [2]; understanding that a simple game is invariant-trade robust if and only if it is k-invariant-trade robust for all positive integer k, and that a simple game is k-invariant-trade robust if a sequence of k or fewer (not necessarily distinct) shift-minimal winning coalitions can never be rendered losing by interchanging k players.

The first advantage of the method we propose is that the set of shift-minimal winning coalitions is a subset of winning coalitions. Thus, we only need to consider lists of shift — minimal winning coalitions to check whether a simple game is weighted. This means a considerably reduction with respect to the standard method which considers all allowable winning coalitions.

On the other hand, non-complete simple (or non-swap robust) games are non-weighted. Thus we can omit them and be restricted to the class of complete (or swap robust) games in order to find out all non-isomorphic weighted games. A classification theorem for complete simple games [1] allows to work with models of coalitions instead of coalitions. Hence, we only need to consider lists of models of shift-minimal winning coalitions to test invariant-trade robustness. This constitutes the second advantage.

The likelihood of our method is tested by a computational implementation. For instance, Taylor and Zwicker [5] proved that there exists simple games which are 2-trade robust but not 3-trade robust for 9 players. We check that *all* simple games with less than 9 players are either weighted or not 2-trade robust. Thus, we have proven that 9 is the *minimum* number of players for a simple game to be 2-trade robust but not 3-trade robust. We also classify *all* complete simple games with less than 9 players according to they are weighted or to the maximum degree of achieved invariant-trade robustness. For example, for 6 players there are 3 complete simple games which are 2-invariant-trade robust but not 3-invariant-trade robust, for 7 players there are 14 complete simple games which are 3-invariant-trade robust but not 4-invariant-trade robust, for 8 players there are 70 complete simple games which are 4-invariant-trade robust but not 5-invariant-trade robust.

In general, Taylor and Zwicker proved [5] that there are simple games, with k^2 players, which are (k-1)-trade robust but not k-trade robust. Similarly, we have proven the existence of simple games, with $2k$-1 players, which are (k-1)-invariant-trade robust but not *k*-invariant-trade robust. Moreover, these sequences of games are achieved by considering games with only three types of equivalent players.

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Hierarchies Achievable in Simple Voting Games

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Keywords: *Hierarchy, Power indices, Simple game, Weak desirabilitry relation, Weakly linear simple games*

A previous work by Friedman, McGrath and Parker [Theory and Decision 61, 305-318 (2006)] introduces the concept of a hierarchy of a simple voting game and characterizes which hierarchies, induced by the desirability relation, are achievable in linear games.

In this paper, we consider the problem of determining all hierarchies, conserving the ordinal equivalence between the Shapley-Shubik and the Penrose-Banzhaf-Coleman power indices, achievable in simple games. It is proved that only four hierarchies are non-achievable in simple games. Moreover, it is also proved that all achievable hierarchies are already obtainable in the class of weakly linear games.

Our results prove that given an arbitrary complete pre-ordering defined on a finite set with more than five elements, it is possible to construct a simple game such that the pre-ordering induced by the Shapley-Shubik and the Penrose-Banzhaf-Coleman power indices coincides with the given pre-ordering.

The aim of this paper is twofold. Firstly, we characterize all achievable hierarchies, induced by the weak desirability relation, in linear simple games and prove that even the hierarchies not achievable in linear simple games are achievable in weakly linear simple games provided that the number of voters is high enough. Secondly, we demonstrate that all hierarchies achievable in simple games are obtainable in weakly linear games. More precisely, we will prove that:

- all hierarchies are achievable in the class of weakly linear games as long as the number of voters is greater than 5.
- all strict hierarchies are achievable in the class of weakly linear games as long as the number of voters is greater than 4.
- Exactly four hierarchies are not achievable in weakly linear games but all of them concern games with less than 6 voters.
- These four hierarchies are not achievable either in the class of all simple games.

Construction of Adaptive Control on the Basis of Methods of Differential Game Theory

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Keywords: *Adaptive control, Differential games, Extremal shift, Maximal stable bridges*

Abstract: *Among practical control problems under disturbances, problems are typical where the level of a dynamic disturbance is unknown a priori. We call a feedback control adaptive if it automatically tunes its level according to the actual level of the disturbance: a "weak" disturbance is parried by a "weak" useful control, and a "strong" disturbance forces applying a useful control of a stronger or even extremal level. In the work, we suggest a method for constructing such a control. The method is based on results from the theory of antagonistic differential games. The quality of the suggested method is shown by simulating a problem of aerial intercept of one weak-maneuvering object by another (of a missile by an anti-missile).*

To construct the adaptive control, we use the theory of antagonistic differential games [1,2] with geometric constraints given for both players. Consider a family of differential games where the geometric constraint for the second player's control depends on a scalar parameter. A constraint for the first player's control and, therefore, a stable bridge in the game space are connected to each value of the parameter too. Changing the parameter, we form a family of bridges, which is ordered by inclusion with increasing the parameter. The first player guarantees keeping the system in a tube using his control of the corresponding level if the second player's control obeys its corresponding constraint. This family allows us to construct a feedback control of the first player and to compute the guarantee provided by this control.

Assume that the system is influenced by a disturbance, which does not exceed some level. Then the system motion will cross the bridges from the family until it reaches (from above or below) the boundary of a bridge, which corresponds to this level of the disturbance. Further, the motion will come inside the bridge. So, the system tunes automatically the level of the useful control to the actual, but unknown level of the disturbance. Due to this, the control is called adaptive.

The idea described above is quite general. Its concrete realization depends on the opportunity to realize an algorithm for constructing stable bridges for systems of certain types.

In the work, we consider problems with linear dynamics, fixed terminal time, and convex compact terminal set, to which the first player tries to guide the system at the terminal instant. The useful control is bound by a geometric constraint, which is a compact convex set. These properties of the system allow us to form quite easily the family of stable bridges and the adaptive control. Namely, under these conditions, we need to compute some two bridges only [3], which are stored in memory and generate the ordered family. So, on the basis of these two bridges at any time instant, we can construct a suitable bridge from the family. Then the control is produced by the extremal shift [1,2] to this bridge. Efficiency of this algorithm is provided by the fact that all the bridges have convex time sections.

Corresponding software is developed now for the case when the terminal set is defined by two components of the phase vector at the terminal instant.

The quality of the suggested method is shown by simulating a problem of aerial intercept of one weak-maneuvering object by another (of a missile by an anti-missile) [4, 5].

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The Dynamic Game with State Payoff Vector on Connected Graph[1]

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Keyword: *Connected graph, State payoff vector, Simple strategy, Absolute equilibrium, Three-dimensional mesh-like graph*

By introducing state payoff vector to every state node in connected graph in this paper, dynamic game is considered on finite graph. The concept of strategy for games on graphs defined by C. Berge is introduced to prove the existence theorem of absolute equilibrium for games on connected graphs with state payoff vector. The complete algorithm and an example for three-dimensional connected mesh-like graph are given in this paper.

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A Game Theory Analysis of the LNG Market: The scenario Bundle Method

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Keywords: *Game Theory, LNG, Gas Supply, Strategic Decision-Making, Scenario Bundles, Oligopolistic Competition, Early Commitment, Entry Deterrence*

Meeting the world's energy demands is one of the greatest challenges in the 21st century and, in many respects, natural gas is considered as the successor of oil. The Liquefied Natural Gas (LNG) trade is without doubt one of the most interesting areas in energy shipping, which dominates the world bulk maritime transport. Indeed, while for many decades natural gas markets were localized and isolated, the LNG trade (that is the transport of natural gas by sea) has contributed to the development of a "world gas market".

In the context of the present research, game theory concepts are used to analyse strategic decision-making for actors involved in the LNG market. This presentation will focus on a "scenario bundle" approach to LNG and the energy markets. Reference will also be made to other game theoretic contributions of the present research to the analysis of the LNG market.

A scenario bundle analysis of the Greek market's security of gas supply

The scenario bundle method was developed by Selten (1999) to address international conflict situations. It will be applied to examine the natural gas security of supply of the Greek market. As Selten explains, this approach is a semi-formal, rather than mathematical, game theoretic modelling approach. Scenario bundles are simple game structures and a systematic way of using qualitative judgments as a basis for the construction and evaluation of scenarios. The modelling task requires that the following parameters are examined:

Assumptions: The scenario bundles are topical models as they relate to a specific region at a specific point of time. In this application, the supply of the Greek gas market is considered within an indicative future period of 5 years. Scenario bundles indicate possible future developments. The method does not promise predictive reliability or moreover success. No method exists to claim a certainty about future strategic decision-making of competing actors in any market. However, scientific speculations about future developments are not deterred by the lack of predictive reliability. The method suggests that a systematic procedure for the integration of judgments may achieve better results than the unaided intuition of well-informed observers, market analysts or players (Selten, 1999).

The actors are assumed to be rational decision makers. Each player's objective is to maximize the expected value of his own payoff, measured in some utility scale (Myerson, 1991, Luce & Raiffa, 1989).

Actors and Goals: The actors or players of the game need to be defined. In this application, the players are Greece, its pipeline suppliers Russia and Turkey, the LNG supplier Algeria and a possible new LNG supplier. Also the transit countries Bulgaria, Romania, Ukraine (for the supply of Greece from Russia) are grouped in one transit player.

Goals are defined for each of the players. The goal is a basic datum of an actor's rational decision making. In this setting, primarily strategic commercial goals are considered (political/other goals could be introduced, but will be avoided in this presentation). For Greece, the goal is to secure its continuous supply with gas. Russia's goal is to maintain or increase its export volumes to Greece and maintain its predominant role in the area as a gas supplier. Turkey's goal is to establish its role in the area as a gas transit supplier and to the West through Greece. Algeria's goal is to maximise its exports to Greece. The unknown LNG supplier's goals are also considered as commercially driven. The transit player's role should be considered minor, except in crisis situations or under the influence of other events.

Initial options: Applications of the scenario bundle method start from a situation in a specific geographical area at a specific point in time. Initial options are options which are open at the initial situation, before anything else has happened. The examined scenario bundle starts with a cut-off of gas supply to Greece from Turkey.

Scenario bundle construction: The graphical representation by a game tree is a natural way to describe a scenario bundle. Scenario bundles are actually extensive games. The initial situation corresponds to the origin of the tree, the starting point. The origin is a decision point for a player, as a result of an initial option which generates the scenario bundle. Possibilities are represented by branches of the tree leading to different nodes (as many branches as the options). Supposing that the initial option has been taken, it is examined which actors will need to react and make a decision. Accordingly the tree is continued. Choices of players which are “strategically” taken at the same time (not necessarily in real time terms) are graphically indicated.

Stopping principles: An end-point is a node beyond which the construction of a scenario bundle is not continued. The stopping principles put an end to the construction of a scenario bundle, which could otherwise continue indefinitely. According to Selten (1999), the construction of a scenario bundle is continued until a blind alley end-point, an inferiority end-point or a normal end-point.

A scenario bundle ends at a blind-alley end-point, when no plausible options can be found after it. A scenario bundle ends at an inferiority end-point, when at that node an alternative option to a certain one will not be taken, no matter what reactions may be expected afterwards. The construction of the scenario bundle does not continue after an inferior alternative. A scenario bundle arrives at a normal end-point when a node without reactive pressure is reached. A node with reactive pressure on the contrary is a node where a player or a group of players are under pressure to make decision whether to react or not. Generally, a normal end-point could be seen as a new initial situation with a variety of new scenario bundles beginning there.

Judgments and analysis: The combined process of analysis and preference judgement begins at the end of the bundle and proceeds backwards. In this way, equilibrium solution are determined. During this backward process, choices which are judged not to be preferable are crossed out. An equilibrium solution is a collection of choices not crossed out.

Other parameters: Influential factors (external to the players) may play an important role in the evolution of a scenario bundle. For example, a gas market crisis in the broader area, initially not directly involving the interaction of players, or the development of gas supply projects peripherally to the examined area.

A coalition is a group of players which cooperate in order to take a common action. Such possible and plausible coalitions should be identified.

The plausibility of initial options as well as of reaction options should be tested using certain criteria. In this sense, options should be realistic (realism criterion) and desirable for players (desirability criterion).

Consequences: They are the pay-offs of the game. The examined scenario bundle reaches some interesting conclusions for both the Greek strategy for security of gas supply, as well as for the Russian export strategy.

Benefits: According to Selten, the benefits of the method can be understood by considering an analogy with a chess player. A chess player who tries to plan ahead cannot predict the future course of the game, however he approaches his decision problem with a "predictive spirit" and he explores the likely consequences of a selection of plausible moves. All possibilities cannot be examined, so a selection has to be made using criteria of plausibility. Stopping principles also need to be used to limit the depth of explorations. In the end, such an analysis is necessary in order to play in an efficient way and hope for success. In this sense, human decision making in chess seems to be analogous to the construction and evaluation of scenario bundles.

Other game theory contributions to the analysis of the LNG market

The LNG market is experiencing a tremendous growth. Yet, investments are capital intensive and relatively few and large players are able to enter and stay in the market. Consequently, the decisions of a market player are likely to influence to a significant degree the position of other players; therefore strategic decision-making is crucial at this stage. Because of its distinctive idiosyncrasies, methodologies applicable to other shipping markets fail to support decision-making in the LNG business. The LNG market context is appropriate for the adoption of a (non-cooperative) game theoretic analysis framework. What is important is to anticipate the reactions of competitors, as these may have a direct impact on the value of the firm.

Reference is made to the following topics:

1. Competition over the capacity supplied to the market by the shipowners / LNG shipping companies and over the price they charge for their services. Specifically, the Cournot and Stackelberg models provide some useful intuition regarding capacity competition in LNG shipping. The competing companies are strategic substitutes and a

first main suggestion is that they must take into account the capacity each one supplies to the market. The respective model can provide an aggregate indication of the optimal supply of capacity. The Bertrand model could address price competition in LNG shipping, when market conditions favour interaction on such terms. In this case, the competing companies are strategic complements.

2. The possibility of non-cooperative collusion in oligopolistic competition in the LNG shipping business, as the equilibriums suggested in the previous models are Pareto inefficient. If companies face one-off competition, collusion seems unlikely. However, if they face continuous interaction, non-cooperative collusion can be sustained. A remark of potential commercial value has to do with the need to detect deviation from collusion when it occurs.

3. Early Commitment is related to the rationale that may justify an early strategic investment commitment (it may also be encountered as "pre-commitment"). The value of investments can be under- (or over-) estimated if one considers only their direct effects, while neglecting their strategic effects. Moreover, it is important to know when to compete aggressively and when to coordinate actions with rivals.

4. Entry Deterrence, which is a strategic interaction between a monopolist LNG shipowner in a specific market (the incumbent) and a potential entrant or entrants in that market. The entrants offer identical or close substitute products (services), i.e. the LNG cargoes shipping service. If they enter the market, then the incumbent's profits are reduced, so the incumbent - monopolist tries to prevent other firms from entering the market.

Game theory can be a useful supplement to the intuition of market players, as it helps in identifying right strategies given certain conditions. This is a required step towards making right decisions.

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Quality Competition: Uniform vs. Non-uniform Consumer Distribution

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Keywords: *Duopoly, Non-uniform distribution, Vertical product differentiation, Two-stage game, Nash equilibrium*

Abstract: *We present results of an investigation of a two-stage model of duopoly under vertical product differentiation, when at the first stage companies select quality level and at the second stage they compete in product price. We comparatively analyzed solutions in case of uniform and an non-uniform consumers' distributions. In the model we received the unique Nash equilibrium in an explicit form. The results depend on the initial parameters which characterized the industrial market. Some quantitative simulation examples are given.*

Two-stage game-theoretical model of duopoly under vertical product differentiation is investigated. It is assumed, that there are two firms on some industrial market which produce homogeneous product differentiated by quality. As well we suppose that a quality range is defined. We studied a two-stage model of duopoly, when at the first stage companies define quality level and at the second stage they compete in product price.

It is suggested that individual consumer demand is equal to one. Consumer's utility function when buying a product of quality s of price p is defined as

$$U_\theta(p) = \begin{cases} \theta s - p, & p \le \theta s, \\ 0, & p > \theta s. \end{cases}$$

We call θ the “inclination to quality”, which indicates customer’s willingness to pay for quality. We suppose that θ is uniformly distributed with unit density over the interval $[0,1]$. The market is proposed to be partially covered.

The profit function of the firm i which produces the product of quality s_i, where $s_i \in [\underline{s}, \bar{s}]$, is the following:

$$\Pi_i(p,s) = p_i(s) D_i(p,s) - C, \; i = 1,2,$$

where p_i – product price of the firm i, $p = (p_1, p_2)$ – a vector of products prices of the competitors, $s = (s_1, s_2)$ – a vector of product qualities, $D_i(p,s)$ – the demand function for the product of quality s_i, which is specified, $C = const$ – total costs considered to be constant for both firms.

In the model we received the unique Nash equilibrium in an explicit form. The results depend on initial parameters which characterized the investigated industrial market. Some quantitative simulation examples are given.

This model was extended to the case when consumers are distributed non-uniformly. The comparative analysis of results in the case of uniform and non-uniform consumers’ distribution is presented.

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Game-theoretical Models and Methods of the Organizational Systems Control Theory[1]

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Keywords: *Game theory, Management, Organizational systems control*

Abstract: *Theory of organizational systems control is briefly surveyed – the school that investigates mathematical models of organizational control mechanisms (the decision-making routines). A general framework for an organizational control problem is set; organizational control methods are classified; the basic model of an organizational system is described, and its possible extensions are outlined. The role of game-theoretical models in setting and solving organizational control problems is shown. A classification of game-theoretical models currently used in organizational systems control is proposed, a few examples are given. Perspective trends of game theory development in the context of the organizational systems control theory are outlined.*

1. Introduction

The theory of organizational systems control studies mathematical models of functioning an organization and mechanisms of organizational control (decision-making routines). It is applicable to a wide range of organizational systems – from a particular department, firm or bureau to a region or a country as a whole.

The foundations of the theory of organizational systems control ("active systems theory") were developed in the 70s of the XX century in the works of soviet researchers V. Bourkov, V. Kondratyev, and others [1].

The theory of organizational systems control combines the methods of classic optimal control theory and system analysis with the techniques of operations research, decision-making theory, and game theory. The approach of the theory of organizational systems control is similar in spirit to the theory of the firm and the theory of markets under asymmetric information (especially, contracts theory and mechanism design [9, 11]).

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An objective of the report is to illustrate the role of game-theoretical models in setting and solving organizational control problems. We introduce a classification of game-theoretic models presently used in organizational control theory and outline the perspectives of game theory in the context of organization control theory.

2. Games and mechanisms of organizational control

According to currently accepted methodology [14] an organizational system is defined by:

1. its staff (a set of members of the system);

2. structure (command structure and other relations between members: informational, technological, etc);

3. feasible actions sets for the members of the system;

4. preferences (objectives) of the members;

5. information awareness of the members (information about significant internal and external variables);

6. operation procedure (sequence of decision-making).

Organizational control is understood as an impact on the controlled system for the purpose of maintaining its desired behavior. Control may affect any of the six elements of the organization system enumerated hereinabove. So, one may distinguish between the following types of organizational control mechanisms: staff management [8], structure management [12], institutional management (control of the feasible actions and behavioral norms) [14], incentives management (preferences manipulation) [13], informational management (awareness manipulation) [2, 15], and operation procedure management [12, 16].

Game-theoretical models are widely used to design the mechanisms of organizational control [6]. For instance, the study of the basic organizational system model – principal-agent problem – adds up to the analysis of a hierarchical game [4] (normally of type $\Gamma 1$ or $\Gamma 2$). The primary role of game-theoretical models as applied to organizational control problems is to provide the principal (in whose behalf the control mechanism is designed) with the forecast of controlled system response to a certain control action. For instance, in two-tier organizational system consisting of the principal and several agents, the principal forecasts the agents' game outcome given the control action is known. Then she chooses the control action to maximize the minimum of her objective function by the set of all game outcomes forecasted (e.g., the set of Nash equilibria).

Such approach imposes some restrictions on game-theoretical models and concepts used. Existence of solution of the game becomes a criterion for organizational system controllability. Uniqueness of solution is equivalent to the quality of forecast and directly influences the appropriateness of control action chosen.

Up to now the most thoroughly studied areas of organizational control are the incentive mechanisms in complete information framework along with the models of adverse selection and moral hazard. In solving these problems the classical models of non-cooperative [13] and cooperative [5] games are widely applied. However, the long-term principal-agent relation is an intrinsic feature of organizations. Thus its analysis (for instance, modeling of adaptation effects) demands using the models of dynamic games (see [16] for reference). Herewith the well-known "folk theorems" turn to be the negative results as they enlarge extremely the set of the solutions of the game. The assumption of agents' bounded rationality (i.a. of agents' improvidence) gives one of possible workarounds for this problem.

Game-theoretic models play a minor part in the problems of staff and structure management. They make it possible calculating the operational effect under a certain structure. The ability to write an analytic formula for the effect sufficiently facilitates the subsequent solution stage when the most effective structure (or staff) must be chosen from the enormous set of feasible ones [8, 10, 12].

An analysis of certain game-theoretic models of control (e.g. the extensions of classical Downs model of voting [3]) is hindered by the absence of Nash equilibria in pure strategies. The investigation of such games required the development of a variety of special solution concepts (see [7]).

The models of informational management also demanded the elaboration of new game-theoretic models – the so-called "reflexive games" [15]. In them common knowledge about environment is replaced by information structure – a tree of subjective beliefs. The original concept of informational equilibrium is taken as a basis for the mechanisms being developed for active forecast and informational regulation [2]. The area of current interest is dynamics of agents' informational structures when new information arrives.

3. Conclusions

Thus, the following lines of game-theoretical research seem perspective from the organizational control theory's point of view: the elaboration of refined solution concepts and investigation of dynamic games in the presence of bounded rationality (specifically, in the absence of common knowledge). The solution concepts are also desired to be simple enough to yield the analytical solution of game when a problem is stated analytically.

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How to Play in Macroscopic Quantum Game?

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Keywords: *Quantum games, Non distributive lattices*

Quantum games are usually considered as games with strategies defined not by the standard Kolmogorovian probabilistic measure but by the probability amplitude used in quantum physics. The reason for the use of the probability amplitude or "quantum probabilistic measure" is the non-distributive lattice occurring in physical situations with quantum microparticles. In our paper we give examples of getting non-distributive orthomodular lattices in some special macroscopic situations without use of quantum microparticles. Mathematical structure of these examples is the same as that for the spin one half quantum microparticle with two non-commuting observables being measured.

So we consider the so called Stern-Gerlach quantum games. In quantum physics it corresponds to the situation when two partners called Alice and Bob do experiments with two beams of particles independently measuring the spin projections of particles on two different directions In case of coincidences defined by the payoff matrix Bob pays Alice some sum of money. Alice and Bob can prepare particles in the beam in certain independent states defined by the probability amplitude so that probabilities of different outcomes are known. Nash equilibrium for such a game can be defined and it is called the quantum Nash equilibrium. The same lattice occurs in the example of the firefly flying in a box observed through two windows one at the bottom another at the right hand side of the box with a line in the middle of each window. This means that two such boxes with fireflies inside them imitate two beams in the Stern-Gerlach quantum game. However there is a difference due to the fact that in microscopic case Alice and Bob freely choose the representation of the lattice in terms of non-commuting projectors in some Hilbert space. In our macroscopic imitation there is a problem of the choice of this representation (of the angles between projections). The problem is solved by us for some special forms of the payoff matrix. We prove the theorem that quantum Nash equilibrium occurs only for the special representation of the lattice defined by the payoff matrix. This makes possible imitation of the microscopic quantum game in macroscopic situations. Other macroscopic situations based on the so called opportunistic behavior leading to the same lattice are considered.

About Pontryagin's Direct Methods in Linear Differential Games[1]

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Keywords: *Dynamic games, Numerical methods*

The important contribution to creation and development of the theory of differential games have brought the schools of L.S.Pontryagin, N.N.Krasovskii, B.N.Pshenichnii, L.A.Petrosyan and others scientists. Among L.S.Pontryagin's works under this theory we shall note [1] - [3]. In these papers two direct methods for the solution of linear differential pursuit–evasion games were developed.

The role of these methods in others (non-Pontryagian) formalizations of the theory of differential games was made clear in N.N.Krasovskii, B.N.Pshenichnii and their followers works. We shall note that Pontryagin designs appear useful as well in the consideration of the linear differential games with the fixed duration and with a terminal payment.

This report is devoted to the discussion of the designs of Pontryagin's methods from the computing point of view. We shall note, for example, that both methods of Pontryagin are based on repeated calculation of a geometrical difference (Minkowski difference) of some sets and on calculation of integrals from multiple-valued functions (in accordance with Auman and Riemann). At a practical realization of these operations

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arises the necessity in their regularization. Further, the problem of the approximate calculation of the Pontryagin's alternated integral is interesting. Other computing problems arise as well.

In the report the results of the long-term research on Pontryagin's direct methods conducted on the department of Optimum Control of CMC faculty of the Moscow State University will be reflected. L.S.Pontryagin was the first head of this department and the founder of the education system of CMC MSU in dynamic game theory, numerical methods and applications.

In 2008 we commemorate the 100th Anniversary of Lev Semenovich Pontryagin birthday. Our report is dedicated to this famous event.

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Computer Realization of J. Nash Agencies Method for n-person Game

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In most of the papers devoted to coalitional game theory it is supposed that coalitions are given and not always the problem of coalition formation is discussed. J.Nash proposed a method for coalition formation developed for 3-person games based on the existence of a leader in each coalition[1]. Using J.Nash approach we made a slight modification for n-person game and developed a computer program on "Visual Studio C++". Consider cooperative n-person game with given characteristic function. We have n-players (1, 2, 3,…, n) and 2^n-1 values of characteristic function computed for each coalition. With probability 1/n! the order of players is selected.

Phase 1: With given order, each player selects a partner. A pair (i, j) is called admissible if there exist a pare (j,i). Admissible pairs are called familiar they consist from same elements. With probability ½ select one from two (X_n, Y_n), (X_n, Y_n) familiar pairs (X_n, Y_n), X_n is marked as leader. Then with previously defined order players without pare Z_n and leaders X_n select a partner. If the admissible selection took place the coalition formation is continued in the same manner.

Remark: What to do if the large coalition joints with a small coalition? We suppose that in this case if is fair to take as leader a leader of large coalition.

Phase2: Payoff allocation.

If on same stage formed coalitions do not create admissible pair the game stops and leader of each coalition distributes the value of characteristic function (payoff) between the members of corresponding custom.

[1]http://www.math.princeton.edu/jfnj/texts_and_graphics/AGENCIES_and_COOPERATIVE_GAMES/Recent_Presentation_Versions/Stony.Brook.2007/Workshop_on_Experimental_Economics/stony_brook07_011.b.ppt

Coalition Formation in Super Additive Games

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Keywords: *Airport problem, Coalition, Cooperative game, Shapley value*

Forming a coalition and playing a cooperative game has some profits for all the players participating in the game, but in real world coalitions form hardly or it takes a long time to form. One of the most important reason for this fact is that although all the player are aware of the profits of forming a coalition but they usually do not agree on how to distribute the benefits of the cooperation among them selves immediately. They usually put a lot of time and effort on bargaining to agree on how to do it. This process gets more complicated and difficult to settle down as the number of players increases. This leads the players to have less incentive to cooperate. Although there are some solution concepts for the problem of how to distribute the benefits of a cooperative game among players such as Shapley value and Nucleolus, but this concepts just can settle down by a governor and it is not done in a usual cooperative game in which there is no government and so the bargaining power or other kinds of players power determine the distribution structure.

Another reason describing not forming coalitions with more than a specified number of members in super additive games is that it is more profitable for the most powerful players – in this paper we consider Shapley value of a player as her power – to form a coalition by themselves – which is also easier to form rather than forming by many of the players – and then charge the other players for using the product of their cooperation. Here we have considered “Airport Problem” and we have shown this claim. Further more, in the case that the coalitions with two players forms, by the assumption that each player has the budget and technology to build the band for herself or those airplanes who have a smaller cost of band construction than hers, we have found the coalition which will form independent of the cost structure which can be linear, concave or convex. If we name players as such that, the coalition which forms will be.

In addition the optimum charge fees for the players who are not participated in the coalition are determined so that they prefer the choice of paying the charged fee rather than to form another coalition. We have shown that these charged fees are much more than what they should pay if the grand coalition or coalitions with more players form. As a result, the most powerful players will not participate in the grand or even coalitions with many players because they can have more profit by not cooperate by others, so the grand coalition or coalitions with many players will lead to less profit for the most powerful players if the distribution rule for the profit of cooperation be Shapley value. Hence, Shapley value rule of distribution the profit of cooperation does not leads to form a stable coalition in superadditive games especially in Airport Problem.

Non-Cooperative Policy Rules in an Integrated Climate-Economy Differential Game with Climate Model Uncertainty

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Keywords: *Differential game, Feedback Nash equilibrium, Structural uncertainty, Uncertainty aversion*

Climate-change policy is subject to fundamental uncertainties concerning the underlying scientific information available. Policy makers' decision to take or not take measures today are based on scientists' projections, generated by computer climate models and evaluated for different emissions scenarios. There is consensus within climate research that current data from underlying physical processes are not sufficient to predict future climate sensitivity, being conclusive for how a change in atmospheric CO_2 affects steady state mean atmospheric temperature.

In this paper we analyze this situation by introducing unknown probability distributions of climate sensitivity in an integrated assessment differential game. A set of climate models are generated by perturbing a continuous-time version of the well-known climate model in Nordhaus (1992) and Nordhaus & Yang (1996), making it statistically impossible for players (policymakers) to infer correct future probability distributions about climate sensitivity by using current data. These conditions of uncertainty better describe the real conditions that policymakers today actually are facing in climate-change policy. The structural uncertainty was embedded in an integrated assessment differential game. An analytical feedback Nash equilibrium with time consistent policy rules is defined for N policymakers. The advantage of an analytical solution, compared to numerical simulations, which in general have poorer reliability, is that it allows for a deeper understanding of the solution and opens up for further analyzes on cooperative structures in this game, fulfilling conditions for individual rationality and time-consistent payment streams as defined in e.g. (Yeung & Petrosjan (2004) and (2006)).

Endogenous Communication and Tacit Coordination in Market Entry Games - An explorative experimental study

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Keywords: *Communication, Coordination, Market Entry*

This paper explores the effects of communication in market entry games experimentally. It is shown that communication increases coordination success substantially and generate inferior outcomes for consumers when market entry costs are symmetric. Such effects are not observed when costs are asymmetric, since asymmetries provide a tacit coordination cue used by experienced players (as a substitute to communication). It is also shown that although communication is used both to achieve market domination equilibria and cooperative market separating equilibria, the latter type of communication is much more common and successful.

Adaptive Control in Three-Dimensional Linear Systems with Dynamical Disturbance of Unknown Level[1]

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Keywords: *Adaptive control, Differential games, Maximal stable bridges*

Abstract: *The construction of adaptive control in two-person linear differential games with a three-dimensional terminal set is considered. The level of constraint on the second player's control is not known in advance. The algorithm is based on the construction of the system of maximal stable bridges for conventional differential games with prescribed constraints on the controls of both players.*

Based on methods of the differential game theory [1], a way for the construction of adaptive control was suggested in [2] for problems with an unknown level of dynamical disturbance. The method is applied to systems with linear dynamics, fixed terminal time, and convex terminal set. It is assumed that the useful control is scalar and constrained in modulus. The aim of control is in getting the phase vector onto the terminal set at the terminal instant and as close to its "center" as possible. The dynamical disturbance is also assumed to be scalar and constrained, but the level of this constraint is not known in advance.

The adaptive control is constructed based on the family of stable bridges [1], and each bridge represents a tube in the space *time*×*phase vector* and corresponds to some value of a numerical parameter. The family of tubes is ordered by inclusion with increasing the parameter. An antagonistic differential game with geometrical constraints on the first and second players' controls and its own terminal set corresponds to each value of the parameter. In the original game, the first player is identified with the useful control, and the second player is identified with the disturbance. The property of stability

[1] The work was supported by RFBR, projects nos. 06-01-00414, 07-01-96085

allows the first player to hold the system motion inside each tube under the corresponding level of the disturbance.

Let a disturbance not exceeding some level act onto the system. Then, if the adaptive control is used, the controlled system moves along the considered family of bridges up to the bridge that corresponds to this level of the disturbance. After this, the motion does not fall outside the boundary of this bridge, i.e., it does not go over to the larger bridges. Thus, the system automatically adjusts the level of the useful control to an unknown level of the disturbance.

In realization of the control method suggested in [2], the main difficulty is in the ability of constructing the bridges and forming the system of these bridges. In [2], an algorithm is described that allows us to do this for the case when the terminal set is completely defined only by two components of the multidimensional phase vector of the original linear system. In this case, the passage is possible to the equivalent constructions in the space *time*×*two-dimensional phase variable.* Therefore, the phase space in these constructions is two-dimensional.

This presentation is devoted to the algorithms of constructing a system of embedded stable bridges and the corresponding adaptive control for the case when the terminal set in the original problem is defined by three components of the phase vector. The principal complication is concerned with the constructions in the three-dimensional phase space of equivalent variables.

For each stable bridge, any its *t*-section (here, *t* is a time instant) represents itself a three-dimensional convex body that is approximated by a convex polyhedron. The bridge is constructed by means of the backward procedure on the given grid of the time instants $t_0, \ldots, t_n$. The rule of the passage from the section at the instant t_{i-1} to the section at the instant t_i, where $t_i < t_{i-1}$, is connected with operations of computation of the algebraic sum of the polyhedrons and their intersection. The main ideas of the computational algorithm for such operations are taken from [3].

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Random Priority Two-person Full-information Best Choice Game with Disorder[1]

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Keywords: *Best-choice problem, Disorder, Full-information best choice, Random priority*

Abstract: *The following version of the two-player best-choice problem is considered. A production system is working in the GOOD state and there is a constant probability that it falls into the BAD state (and remains there) at the disorder moment. In the GOOD state system produces i.i.d. r.v. uniform in* $[0,1]$. *In the BAD state system produces i.i.d. r.v. uniform in* $[0,b]$ *(* $b<1$ *). The true state of the system is unknown.*
Two players observe a sequence of i.i.d. r.v. at each time $t=1,2,...,n$ *and decide either CONTINUE (i.e. reject observation* X_t *and observe another) or STOP (i.e. accept and receive observation* X_t *). The objective is to maximize the expected net value of the accepted observation. Recall is now allowed. If both players want to accept observation then a random assignment mechanism is used. A class of one-threshold strategies (i.e. every player set threshold* q *and accept the first observation that greater than* q *and rejecting all observations that less than* q *) is obtained. The asymptotic behavior of the solution is also studied.*

The following version of the two-player best choice problem is considered. A production system is working in the GOOD state and there is a constant probability α that it falls into the BAD state (and remains there) at a disorder moment. The transition matrix is as following:

	G	B
G	α	1-α
B	0	1

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In the GOOD state system produces independent identical distributed random variables (iid r. v.) uniform on [0,1]. In the BAD state system produces iid r. v. uniform on $[0,b]$ ($b \in [0,1]$ is a parameter).

Two players (Player I and Player II) observe the output X_t of the system at each time $t = 1,2,...,N$ and decide either CONTINUE (i.e. reject X_t and observe X_{t+1}), or STOP (i.e. accept and receive X_t). Recall is not allowed (i.e. the observation once rejected cannot be recalled later). Each player knows parameters α and b, but the real state of the system is unknown. The aim of the players is to maximize the expected value of the accepted observation during the given finite period of time N.

When some player accepts an observation at time n, then the other one will investigate the sequence of future observations having an opportunity to accept one of them. The players cannot accept the same observation at the same moment. When both want to accept such an observation a random assignment mechanism is used. This mechanism is defined by the lottery described by a random variable ξ with uniform distribution on [0,1] and a number p ($p \in [0,1]$ is a parameter). If both players wish to select the observation at the same moment then Player I benefits if $\xi \le p$; otherwise Player 2 gets the observation. One can say that the players have random priority to accept an observation. A class of one-threshold strategies (i.e. each player sets threshold q – q_1 and q_2 respectively – and accepts the first observation that greater than q and rejecting all observations that less than q) is studied.

A class of suitable strategies and a gain function for the problem is constructed.

The asymptotic behaviour of the solution is also studied. We propose to find the limit (as $N \to \infty$) decision of the task and numerical results for different values of parameters α, b and p.

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A Characterization of Convex Games by Means of Bargaining Sets

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Keywords: *Bargaining set, Convex game, Cooperative game, Weber set*

Abstract: *The aim of the paper is to characterize the classical convexity notion for cooperative TU games by means of the Mas-Colell and the Davis-Maschler bargaining sets. A new set of payoff vectors is introduced and analyzed: the max-Weber set. This set is defined as the convex hull of the max-marginal worth vectors. The characterizations of convexity are reached by comparing the classical Weber set, the max-Weber set and a selected bargaining set.*

In this paper we shall be concerned with cooperative games with transferable utility, (N,v) where N is the set of players and v the characteristic function of the game. Convexity is an important notion for cooperative game theory since it has been used in many applications. A game is *convex* if, for all $i \in N$,

$$v(S \cup \{i\}) - v(S) \leq v(T \cup \{i\}) - v(T),$$

for all $S \subseteq T \subseteq N \setminus \{i\}$. Characterizations of convexity have been developed since Shapley (1971) introduced the notion. Ten years were needed to obtain the best-known characterization given in terms of the coincidence of the core and the Weber-set (see Shapley (1971) and Ichiishi (1981)). We now address the question of how to characterize convexity by means of bargaining sets.

Shapley (1971) and Ichiishi (1981) prove that a game is convex if and only if every marginal worth vector of the game is in its core. Weber (1988) introduces the set of all convex combinations of marginal worth vectors of a game v (the Weber set, $W(v)$) and proves that the core of the game, $C(v)$, is a subset of its Weber set. Therefore, the characterization of the convexity of a game in terms of its marginal worth vectors is in fact equivalent to the coincidence of the Weber set with the core.

Maschler et al. (1972) prove that the core of a convex game coincides with the Davis and Maschler bargaining set $\mathrm{M}_1^{(i)}(v)$ (Davis and Maschler 1967). On the other hand, Dutta et al. (1989) prove that, for convex games, the core also coincides with the Mas-Colell bargaining set $\mathrm{M\,B}(v)$ (Mas-Colell, 1989). Nevertheless, these coincidences do not characterize the convexity of a game.

In this paper we prove (see Theorem 1) that, for a cooperative game with non-empty core (balanced game), convexity is characterized by the inclusion of the Weber set in the Davis and Maschler bargaining set or, alternatively, in the Mas-Colell bargaining set.

Theorem 1 *Let v be a balanced game. Then, the followings statements are equivalent:*

1. v is convex,
2. $W(v) \subseteq \mathrm{M}_1^{(i)}(v)$,
3. $W(v) \subseteq \mathrm{M\,B}(v)$.

A natural question is whether balancedness might be dropped or relaxed. For three and four-person superadditive games, it is quite easy to check that coincidence between the Weber set and the Davis and Maschler bargaining set characterizes convexity; for five players or more, it is still an open question. Nevertheless, we can give a general characterization on the whole domain of games by using the max-Weber set ($\overline{W}(v)$) which is defined as the convex hull of max-marginal worth vectors.

Theorem 2 *Let $v \in \mathrm{G}^N$. Then, the followings statements are equivalent:*

1. v is convex,
2. $W(v) \subseteq \mathrm{M}_1^{(i)}(v) = \overline{W}(v)$,
3. $W(v) \subseteq \mathrm{M\,B}(v) = \overline{W}(v)$.

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Journals in Game Theory

GAME AND ECONOMIC BEHAVIORTHEORY REVIEW

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Generating Function for Indexes of Power

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Keywords: *Generating function, Index of power*

Abstract: *We consider game of the weighted voting with number of the players n and with quota q. Each player is a party with number of voters w.*

Discontinuous Value Function in Time-optimal Differential Games

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Keywords: *Differential game, Time-optimal problem, Value function*

In the talk, two-person zero-sum differential games are considered, in which the time until a point reaches a given closed terminal set $M \subset R^n$ is the pay-off functional.

We shall keep at a positional formalization of a differential game described in books by N.N. Krasovskii and A.I. Subbotin [1, 2]. Within the framework of the positional formalization, an important question in studying a differential game is searching for a value function, which at each point of the state space of the system matches an optimal guaranteed result in a game starting from this point. Optimal feedback control strategies can be constructed on the basis of the value function.

In general, the value function of a differential game can be non-smooth and discontinuous and can take the improper value of ∞.

Suppose a certain test function to be constructed, and it is required to proof that namely this function is the value function of the game. This problem is closely connected with a characterization of the value function.

A differentiable value function is the unique classical solution of a boundary value problem for a first-order partial differential equation (the Isaacs–Bellman equation) [3].

If the value function is not smooth but continuous, then the concepts of continuous *u*- and *v*-stable functions [2, p. 145], which were introduced into the theory of positional differential games, become fundamental in its characterization. In this case, *u*-stable (*v*-stable) functions with a corresponding boundary condition majorize (minorize) the value function of a differential game, and this is the unique function possessing the properties of *u*- and *v*-stability. Let us remark that the notion of *u*-stable (*v*-stable) function corresponds to the notion of upper (lower) viscosity solution of the Isaacs–Bellman equation (see, for example, [4]).

The characterization of a discontinuous value function is rather complicated [4, 5] and coincides with the notion of discontinuous minimax solution [5, p. 223] of a boundary value problem for the Isaacs–Bellman equation. That is in time-optimal problems, the value function is a unique lower semicontinuous *u*-stable function satisfying a null boundary condition on the boundary of the terminal set, to which the sequence of upper semicontinuous *v*-stable functions converges pointwise. The *v*-stable functions satisfy the same boundary condition and are continuous on the boundary of *M*. Verification of existence of such a sequence and, especially, its construction, are difficult even in the case of the solution of problems in the plane.

In the talk, a theorem on sufficient conditions for a test function to be identical with the value function of the time-optimal differential game to be investigated is formulated, which involves the case of discontinuous value function. Conditions of the theorem require to verify properties, which are similar to properties of a discontinuous minimax solution but in arbitrarily small neighborhoods of subsets, in which boundaries of level sets of the test function are decomposed. In many cases, consideration of several neighborhoods makes the obtained conditions be more convenient for a practical verification than a direct use of the definition of a discontinuous minimax solution.

Application of the theorem is illustrated by an example of a time-optimal differential game in the plane with the dynamics of a conflict-controlled material point.

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Rank-Order Innovation Tournaments

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Keywords: *Research tournament, Innovation race, Two-stage game, Incomplete information*

Research efforts and outcomes are generally private information of innovative firms: research inputs are unobservable and the value of innovations is difficult to evaluate. This is the reason why rank-order tournaments are more adequate incentive schemes rather than a conventional contracting [1]. The theory of rank-order tournaments was pioneered by Lazear and Rosen [2] and promoted as optimum labor contracts. The tournaments refer to incentive compensation schemes which pay according to an individual's ordinal rank rather than on output levels. The research competition may be analyzed in a labor tournament framework [3], [4], [5], [6]. The typical model considers a risk-neutral sponsor (commonly governments or private corporations) and a number of (risk-neutral or risk-averse) contestants (such as research teams, startup companies). The contestants are competing to find the "best" innovation. The winner obtains the prize and the losers get nothing in a "winner-take-all" game. The prize is thus awarded by the sponsor on the basis of relative rank rather than on the absolute performance [7]. Following Taylor [1], research tournaments and innovation races differ fundamentally: in tournaments the terminal date is fixed and the quality of innovations vary, whereas in innovation races the date of discovery is unknown and the quality standard is fixed. An innovation tournament belongs to the class of dynamic n-player two-stage games of imperfect information: at the "entry stage" each firm decides whether to participate, at the "contest stage" each contestant decides whether to invest in each period without knowing the rivals' choices. The game is solved by backward induction: the tournament is first solved for given prizes, then the sponsor's expected profit is computed and the optimal prize is deduced. Provided the objective function is quasiconcave, the tournament subgame has a unique symmetric equilibrium in pure strategies [8], [1]. In most innovation tournaments [1], [9] the value of the winner prize is exogenous. In new tournament models [10], research inputs not only the determine the probability of winning but also the value of the winner prize. This contribution is proposing a computational approach to the innovation tournament models for different probability distributions of shocks. The computations are carried out using the software *MATHEMATICA® 5.1* [11].

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Numerical Construction of Nash Trajectories in a Two-Person Non Zero-sum Linear Positional Differential Game[1]

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Keywords: *Nonantagonistic differential game, Numerical methods*

Abstract: *There are various approaches for computation of solutions in differential games, see, e.g., (Krasovskii and Subbotin, 1988; Krasovskii and Krasovskii, 1995; Basar and Olsder, 1995; Kleimenov, 1993). Many of them suggest numeric methods for solution computation. Such algorithms proposed for antagonistic games are, for example, discussed in papers (Isakova et al., 1984; Vahrushev et al., 1987), as well as in other studies of the same and other authors. Comparing to this, there are distinctly less studies concerning nonantagonistic games and they usually deal with linear quadratic games. The present paper describes an algorithm for Nash equilibrium solutions in linear deferential game with geometrical constrains for players' controls and terminal cost functionals of players.*

Let the dynamics of a two-person nonantagonistic positional differential game is described by the equation

$$\dot{x} = A(t)x + B(t)u + C(t)v + f(t)\ ,\ \ x(t_0) = x_0\ ,\ \ t \in [t_0, \theta] \tag{1}$$

where $x \in R^n$ is a phase vector. Matrices $A(t)$, $B(t)$ and $C(t)$ are continuous and have dimensions $n\times n$, $n\times k$ and $n\times l$ respectively. Controls $u \in P \subset R^p$ and $v \in Q \subset R^q$ are handled by Player1 and Player2 respectively. Here sets P and Q are assumed to be compact polyhedrons. The final time θ is fixed.

Let the goal of Player1 be the maximization of the cost functional $\sigma_1(x(\theta))$ while Player2 maximizes the cost functional $\sigma_2(x(\theta))$ where functions $\sigma_1 : R^n \to R^1$ and $\sigma_2 : R^n \to R^1$ are continuous and concave.

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It is assumed that both players know the value $x(t)$ at current moment of time $t \in [t_0, \theta)$. Thus formalization of players' strategies in the game could be based on the formalization and results of the positional antagonistic differential games theory [1, 2]. According to this formalization (see also [3]) the strategies are described belonging to the class of pure positional strategies and are equated to pairs of functions. A strategy of Player1 U is equated to a pair $\{u(t, x, \varepsilon), \beta_1(\varepsilon)\}$ where $u(\cdot)$ is an arbitrary function of the position (t, x) and a positive precision parameter ε and possesses the value in the set P. The function $\beta_1 : (0, \infty) \to (0, \infty)$ is a continuous monotone one and satisfies the condition $\beta_1(\varepsilon) \to 0$ if $\varepsilon \to 0$. The function $\beta_1(\cdot)$ has the following sense. For a fixed ε the value $\beta_1(\varepsilon)$ is the upper bound for the step of a subdivision of the interval $[t_0, \theta]$ used by Player1 for forming step-by-step motions. A strategy $V \div \{v(t, x, \varepsilon), \beta_2(\varepsilon)\}$ of Player2 is defined analogously. A pair of strategies (U, V) generates a motion $x[t, t_0, x_0, U, V]$. The set of motions $X(t_0, x_0, U, V)$ is non-empty.

A pair of strategies (U^N, V^N) is called a Nash equilibrium solution (an N-solution) of the game if for any motion $x^* \in X(t_0, x_0, U^N, V^N)$, any $\tau \in [t_0, \theta]$ and any strategies U and V the following inequalities hold

$$\max \sigma_1(x[\theta, \tau, x^*[\tau], U, V^N]) \leq \min \sigma_1(x^c [\theta, \tau, x^*[\tau], U^N, V^N]),$$
$$\max \sigma_2(x[\theta, \tau, x^*[\tau], U^N, V]) \leq \min \sigma_2(x^c [\theta, \tau, x^*[\tau], U^N, V^N]). \quad (2)$$

The operations of min and max in (2) are taken in the sets of corresponding motions.

It is shown in [3] that finding N-solutions in the problem of the type under consideration could be reduced to solution of a non-standard auxiliary problem of control. In our case this non-standard problem of control can be formulated as follows.

It is needed to find a pair of measurable controls $u(t)$ and $v(t)$, $t_0 \leq t \leq \theta$, guaranteeing the fulfillment of the conditions

$$\gamma_i(t, x(t)) \leq \gamma_i(\theta, x(\theta)) = \sigma_i(x(\theta)), \; i = 1,2 \quad (3)$$

for all $t \in [t_0, \theta]$.

Here the function $\gamma_i : [t_0, \theta] \times R^n \to R^1$ denotes a value function of the following antagonistic differential game Γ_i. The dynamics of the game Γ_i is described by the equation (1). But in this game Player i maximizes his cost functional $\sigma_i(x(\theta))$ and Player 3-i counteracts this goal. It is known [1, 2] that the value function of this game exists and it is continuous.

Trajectories of the system (1) satisfying the conditions (3) are called N-trajectories. It is easy to describe an N-solution which generates an N-trajectory (see [3]) by using so-called punishment strategies.

This report deals with N-solutions in the class of positional strategies. Generally it is very difficult to find the whole set of solutions for the non-standard problem described above. Therefore the report presents an algorithm for constructing only some N-solution that essentially is a constructing of N-trajectories.

The proposed algorithm is based on the following procedure (see [6]). The procedure uses the principle of non-decrease of players' payoffs, maximal shift in the direction best for one and another player and Nash equilibria in auxiliary bimatrix games.

The procedure implies existence of functions $\rho_1(t, x)$ and $\rho_2(t, x)$ such that Player i is interested in increasing the function $\rho_i(t, x)$ along the solution trajectory. In particular, the function $\gamma_i(t, x)$ mentioned above could be chosen as $\rho_i(t, x)$.

Suppose the position (t, x) is given. We fix $\varepsilon > 0$ and put $\tau(t, \varepsilon) = \min \{t + \varepsilon, \theta\}$ and denote by $w^1(\tau(t, \varepsilon))$ and $w^2(\tau(t, \varepsilon))$ the maximum points for functions $\rho_1(t, x)$ and $\rho_2(t, x)$ correspondingly in the $\tau(t, \varepsilon)$-neighborhood of the point x.

Consider vectors

$$s^1(\tau, x, \varepsilon) = w^1(\tau(t, \varepsilon)) - x,$$
$$s^2(\tau, x, \varepsilon) = w^2(\tau(t, \varepsilon)) - x.$$

We define vectors $u_{10}(t, x, \varepsilon)$, $v_{10}(t, x, \varepsilon)$, $u_{20}(t, x, \varepsilon)$ and $v_{20}(t, x, \varepsilon)$ from conditions

$$\max_{u\in P, v\in Q} s^{1T}[B(t)u + C(t)v] = s^{1T}[B(t)u_{10} + C(t)v_{10}],$$

$$\max_{u\in P, v\in Q} s^{2T}[B(t)u + C(t)v] = s^{2T}[B(t)u_{20} + C(t)v_{20}].$$

Now we construct the auxiliary bimatrix 2 x 2 game (A, B) where the first player has two strategies: "to choose u_{10}" and "to choose u_{20}". Similarly, the second player has two strategies: "to choose v_{10}" and "to choose v_{20}". Corresponding payoff matrices are defined as follows:

$$A = \begin{Vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{Vmatrix}, \quad B = \begin{Vmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{Vmatrix},$$

$$a_{ij} = \rho_1(\tau(t,\varepsilon), (x + A(t)x + B(t)u_{i0} + C(t)v_{j0})\tau(t,\varepsilon)),$$

$$b_{ij} = \rho_2(\tau(t,\varepsilon),(x + A(t)x + B(t)u_{i0} + C(t)v_{j0})\tau(t,\varepsilon)) ,$$

i, j = 1,2.

The bimatrix game (A, B) has at least one Nash equilibrium in pure strategies. We take a Nash equilibrium of it as controls of both players within the interval $[t, \tau(t, \varepsilon)]$. Such an algorithm of players' controls construction generates a trajectory being an N-trajectory.

Results of a previous related research of numerical trajectories computation for Stackelberg solutions for the special game subclass can be found in [7]. Value functions $\gamma_i(t, x)$ were chosen as functions $\rho_i(t, x)$. The value function calculation program that was used is based on [4, 5].

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Learning Process and Emergence of Cooperation: The Prisoner's Dilemma Case

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Keywords: *Prisonner's dilemma, Cooperation, Learning*

Abstract: *Doghmi and Kobiyh (2007) provide that if the proportion of individuals having the reciprocal behavior is greater to a certain limit, then the cooperation emerges. We are illustrated the influence of the degree of cooperation and the length of game on the emergence of the cooperation in an evolutionary setting. In this work, we attempt to explain how cooperation emerges using a model based on learning process under the model (EWA).*
We present a learning model in the repeated game of prisoner dilemma and we examine the abilities of learning models to describe subject behavior in experiments. We consider two players, one adopts a behavior incentive to cooperation, the other is a subject to learning. Players observe only the path of game and not the rule of behavior chosen by their opponents. On the basis of this observation, each player must try infer what strategy the opponent has chosen. Cooperation emerge when a couple of state learning and a degree of cooperation are realized.

Traditional game theory assumes that players make optimum plans assuming that everyone else makes optimum plans, and that all of this is common knowledge. In recent years an alternative view has emerged, prompted by experiments with human subjects. The laboratory evidence confirms that people do care about other's payoffs as well as their own (Cox et al., 2007). People surely care about their own material well-being. In some contextes people also may care about other's well-being. Evidence from the laboratory and the field has begun to persuade economists to develop specific models of how and when a person's preferences depend on others' material payoffs (Cox and al., 2008).

According to Fehr and Gachter (2000) many people deviate from purely self-interested behaviour in a reciprocal manner. The received explanation is that players are motivated by reciprocity and desire to reward kindness and punish hostility. Previous works by Rabin (1993); Berg, Dickhaut, and McCabe (1995); Fehr, Gachter, and Kirchsteiger (1997) indicate that concerns may play an important role in hazard contexts. However, these researches did not study how the principals choose between explicit and implicit incentives. Our work addresses the question of how concerns for fairness and reciprocity affect the optimal choice.

To explain these experimental results, we which introduce learning in the game. Generally, two varieties are received the most scrutinyin experiments. In belief learning models, players form beliefs on the basis of opposants' past decisions and tend to play strategies today that have relatively high expected payoffs given those beliefs. In reinforcement learning models, players tend to play strategies that have paid off relatively well in the past. Both models can be combined in flexible hybrid models experience-weighted attraction (EWA).

In experiments one observes that people cooperate much more than predicted by standard economic theory. Doghmi and Kobiyh (2007) provide that if the proportion of individuals having the reciprocal behaviour is greater to a certain limit, then the cooperation emerges. We are illustrated the influence of the degree of cooperation and the length of game on the emergence of the cooperation in an evolutionary setting. In this work, we attempt to explain how cooperation emerges using a model based on learning process under the model (EWA). We present a learning model in the repeated game of prisoner dilemma and we examine the abilities of learning models to describe subject behaviour in experiments. We consider two players, one adopts a behaviour incentive to cooperation, the other is a subject to learning. The idea of this work is to show that a process learning must lead the people to choose gradually kind of behaviour stable and fit for environment.

Economics is based on incentives and it derives its strength from being able to predict how people change their behaviour in response to changes in incentives (Fehr and Falk, 2002). In view of the importance of fairness and reciprocity concerns, it is natural to seek an explanation in the context of fairness and reciprocity models. We specify how each player's beliefs about other's strategies evolve over time, and also how he himself revises his strategy in the wake of his conjectures. We formalise this learning process in terms of a random sequence called *learning sequence*, and we show that if players are motivated by fairness and reciprocity, then equilibrium behaviour accords well with the aforementioned stylized facts. Each sequence is characterized by a state of learning and a degree of cooperation. Players observe only the path of game and not the rule of behaviour chosen by their opponents. On the basis of this observation, each player must try infer what strategy the opponent has chosen. Cooperation emerge when a couple of state learning and a degree of cooperation are realized.

Pareto-optimal Solutions in a Game Theoretical Model of Environmental Management

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Keywords: *Dynamic games, Dynamic programming, Environmental management, Pareto-optimal solution*

Abstract: *In this paper a game theoretical model of environmental management is considered. There are n players which are involved in the differential game of pollution cost reduction. Any player has two types of cost. The dynamics of the game is linear. Each player seeks to minimize a stream of discounted sums of costs. The Pareto-optimal controls and Pareto-optimal trajectory are found by the help of Hamilton-Jacobi-Bellman equation.*

In this paper a game theoretical model of environmental management is considered. Above mentioned model was firstly introduced in Germain et al. (1998) in discontinuous time. Then it was used in the paper of Petrosjan and Zaccour [1] in continuous time. This paper discusses the same model with some distinctions. There are n players which are involved in the game of pollution cost reduction. Let I be the set of players involved in the game:

$$I = \{1,2,\ldots,n\}.$$

The game begins at the instant of time t_0 from the initial state x_0. Denote by $u_i(t)$ emission of player i at time t. Let $x(t)$ be the stock of accumulated pollution by time t. The dynamics of the game is governed by the following differential equation:

$$\begin{aligned} \dot{x}(t) &= \sum_{k=1}^{n} u_i(t) - \delta x \\ x(t_0) &= x_0. \end{aligned}$$

Any player has two types of cost: emission reduction cost and damage cost. Emission reduction cost are incurred by country i when limiting its emission to level $u_i(t)$. The function describing emission reduction cost is equal to

$$C_i(u_i(t)) = \frac{\gamma}{2}(u_i(t) - \overline{u}_i)^2,$$
$$0 \le u_i(t) \le \overline{u}_i, \qquad \gamma > 0.$$

Damage cost depends on the stock of accumulated pollution by the linear way. These functions are different for all players and equal

$$D_i(x(t)) = \pi_i x(t), \qquad \pi_i > 0, \qquad i = 1, 2, \ldots, n.$$

Each player seeks to minimize a stream of discounted sum of emission reduction cost and damage cost. The payoff function of any player equals

$$K_i(x_0, t_0; u_1(t), u_2(t), \ldots, u_n(t)) = \int_{t_0}^{\infty} \rho e^{-\rho(t-t_0)} \{C_i(u_i(t)) + D_i(x(t))\} dt.$$

The Pareto-optimal controls and Pareto-optimal trajectory are found by the help of Hamilton-Jacobi-Bellman equation.

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Solutions for Repeated Bidding Games with Incomplete Information for the Case of Three Admissible Bids[1]

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Keywords: *Bidding game, Incomplete information, Optimal strategy, Recursive sequence of second order, Repeated game*

De Meyer and Saley (2002) demonstrated the idea of strategic origin of the Brownian component in the evolution of stock market prices by help of a simplified model of multistage bidding between two agents for risky assets (shares).

Before bidding starts a chance move determines the realization value of a share once for all. This value is either one with probability p or zero with probability $1-p$. Player 1 is informed about the chance move outcome, Player 2 is not. Both players know probabilities of a chance move. Player 2 knows that Player 1 is an insider.

At each subsequent step $t=1,2,\ldots,n$ both players simultaneously propose their prices for one share. Players may make arbitrary bids. The maximal bid wins and one share is transacted at this price. If the bids are equal, no transaction occurs. Each player aims to maximize the value of his final portfolio (money plus realization value of obtained shares).

We modified this model assuming that players may make only discrete bids proportional to a minimal currency unit, i.e. admissible bids are multiples of $1/m$ (m admissible bids). These models are reduced to two person zero-sum repeated games $G_n^m(p)$ with incomplete information on one side (see Aumann, Mashler (1995)). In

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Domansky (2007), Domansky, Kreps (2007) these repeated games of unlimited duration are solved explicitly.

But the solution of such n -step games is an open problem. Here we solve it for the case $m = 3$. We get the solutions for the games with three admissible bids for any number of steps n . In this case there are only three effective bids, namely 0 , $1/3$, and $2/3$. We get the solutions for the games $G_n^3(p)$ of any finite duration n and for any prior probability $p \in [0,1]$. The values $V_n^3(p)$ of these games and optimal strategies of players are expressed by means of recursive sequence of second order

$$\delta_{n+1} = 2(\delta_n + \delta_{n-1}),\ \delta_1 = 2,\ \delta_2 = 4.$$

The elements of this sequence can be represented analitically as

$$\delta_n = \frac{(1+\sqrt{3})^n - (1-\sqrt{3})^n}{\sqrt{3}}.$$

Theorem. *For any n , the continuous concave piecewise linear function $V_n^3(p)$ has three break points $p_i(n)$, $i = 1,2,3$. The first and the third points do not depend on n : $p_1(n) = 1/3$, $p_3(n) = 2/3$. For $n > 1$, the second break point is given by*

$$p_2(n) = (\delta_{n-1} + \delta_n) / (\delta_{n-1} + 2\delta_n).$$

The values of V_n^3 , $n >\geq 1$, at the break points are

1) $3V_n^3(1/3) = 1 - 2/3\delta_n$;

2) $3V_n^3(p_2(n)) = 1 - 1/(2\delta_n + \delta_{n-1})$;

3) $3V_n^3(2/3) = 1 - 1/3\delta_{n-1}$

with one exception: $V_1^3(2/3) = 2/9$

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Journals in Game Theory

INTERNATIONAL GAME THEORY REVIEW

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Numerical Construction of Singular Surfaces in Linear Differential Games with Elliptic Vectograms

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Keywords: *Differential games, Numerical construction, Optimal motions, Singular surfaces*

Abstract: *The talk suggests an algorithm for effective constructing singular surface for linear differential games with fixed terminal time, geometric constraints for the players' controls and terminal quasiconvex payoff function depending on two components of the phase vector. Results of computation of several model examples are given.*

Singular surfaces of a differential game are surfaces in the game space, where the optimal trajectories have some peculiarities (fractures, splits, junctions). Investigation of singular surfaces is important because they give some “skeleton” of the game formation. The basic idea of the singular surface classification was suggested by R.Isaacs in his book [1]. It is based on the analysis of fields of extremal motions of the system and involves investigation of the areas, which either are not filled by the motions, or contain intersecting bundles of motions. Such an analysis is especially effective for problems with two-dimensional phase vector. In the case of the phase variable of a higher dimension, the type of interaction of optimal motions can be very complicated.

Up to date, there were no generic algorithms for numeric constructing singular surfaces. The aim of the talk is to tell about such algorithms carried out by the author for linear differential games with fixed terminal time, geometric constraints for the players' controls and terminal quasiconvex payoff function depending on two components of the phase vector. These algorithms are based on detection and classification of singular points on the boundary of level sets (Lebesgue sets) of the value function. The points are joined into lines lying on the boundary of the level sets. The lines taken from a collection of level sets give the singular surfaces.

The algorithms for constructing level sets of the value function (maximal stable bridges) were developed in the Institute of Mathematics and Mechanics (Ekaterinburg, Russia) in 1980's. For the case of games with two-dimensional phase vector, such an algorithm [2] produces a collection of polygons approximating time sections of the maximal stable bridge. A procedure for detecting and classifying singular points on the boundaries of these polygons is embedded to the algorithm as a subroutine.

For games with scalar controls of both players, an algorithm for constructing singular surfaces was developed earlier [2]. In this work, main attention is paid to the case of arbitrary strictly convex compact constraints for the players' controls. As one of test examples, a problem of aerial interception is taken, whose singular surface were analytically studied in the work [3]. Also, some specially selected differential games are considered from the class called "Pontryagin's generalized test example". In these problems, the players' vectograms are ellipses. Some tricky combinations of typical surfaces (dispersal, focal, equivocal) are found.

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Three-Sided Matchings and Separable Preferences

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Keywords: *Separable preferences, Stable Matchings, Three-Sided Matchings*

Abstract: *In this paper we provide sufficient conditions for the existence stable matchings for three-sided systems.*

The two-sided matching model of Gale and Shapley (1962) can be interpreted as one where a non-empty finite set of firms need to employ a non-empty finite set of workers. Further, each firm can employ at most one worker and each worker can be employed by at most one firm. Each worker has preferences over the set of firms and each firm has preferences over the set of workers. An assignment of workers to firms is said to be stable if there does not exist a firm and a worker who prefer each other to the ones they are associated with in the assignment. Gale and Shapley (1962) proved that every two-sided matching problem admits at least one stable matching.

In this paper we extend the above model by including a non-empty finite set of techniques. A technique can be likened to a machine that is owned by a technologist who is neither a firm nor a worker, and which the firm and worker together use for production. Further each technologist owns exactly one technique. Each firm has preferences over the set of ordered pairs of workers and techniques, each worker has preferences over the set of ordered pairs of firms and techniques and each technologist has preferences over ordered pairs of firms and workers. Such models [see Alkan (1988)] are called three-sided systems. A matching in a three-sided system consists of disjoint triplets, each triplet comprising a firm, a worker and a technologist. A stable matching for a three-sided system is a matching which does not admit a triplet whose members are better off together than at their current designations. Alkan (1988) provided an example of a three-sided system that does not admit a stable matching. Danilov

(2003) established the existence of a stable matching for lexicographic three-sided systems.

The preference of a firm is separable if its preference over workers is independent of the technique and its preference over techniques is independent of the worker. The preference of a worker is separable if its preference over firms is independent of the technique and its preference over techniques is independent of the firm. A three-sided system is said to be separable if preferences of all firms and workers are separable. Through out the paper, we assume that the preferences of the workers are separable between firms and techniques. A special case of such preferences is lexicographic preferences, with firms enjoying priority over techniques. If in addition the preferences of the firms are lexicographic, with workers enjoying priority over techniques, then the system is called lexicographic. Lexicographic systems are clearly separable.

In this paper we show that if a three-sided system is lexicographic for workers and satisfies a property called *Technical Specialization* then there exists a stable matching. Technical Specialization says: given two distinct firm-worker pairs, the technique that is best for the firm in one pair is different from the technique that is best for the firm in the other. Note that the discrimination property is strictly stronger than the weak discrimination property that we discussed earlier. We also provide an example of a three-sided system with preferences of workers being both lexicographic as well as separable, that does not admit a stable matching. In this example the preferences of the firms are neither lexicographic nor separable.

Neither technical specialization nor the proof of theorem that establishes the existence of a stable matching when technical specialization is satisfied by a three-sided system, takes cognizance of the preferences of the technologists. In a way, the stable matching that is obtained, may have resulted by 'coercing' the technologists. While this may make the technical specialization an unpalatable assumption, it is worth remembering, that a stable matching for a three-sided system, does not require that every side of the system play an active role in determining its viability. Alternatively one may assume that the three-sided system is strongly separable i.e. lexicographic for workers and separable for firms. In such a scenario we need to assume that the preferences of firms and technologists over workers are in "agreement" (i.e. given a firm, a technologists ranks the workers in the same way that the firm does) to show that a three-sided system admits a stable matching. Agreement over workers in a strongly separable environment implies some kind of a hierarchy where the worker cares only about the firm and forms the bottom layer, whereas the technologist's preferences over the workers "echoes" the preferences of the firm it is engaged with.

Following the tradition of Gale and Shapley (1962), we model our analysis in terms of a firm employing at most one worker. By present day reckoning, a firm employing at most one worker, is usually a small road-side shop, rather than an industrial unit. Hence, it might appear as if our analysis has little if no relevance to the more common real world situations. However, it may well be a reasonable starting point for the cooperative theory of multi-sided systems. Roth and Sotomayor (1988) contain an elaborate discussion of matching models, where firms may employ more than one worker. It turns out in their analysis, that the cooperative theory for such firms is almost identical to the cooperative theory arising out of the Gale and Shapley (1962) framework. This occurs, since each firm can be replicated as often as the number of workers it can employ, with each replica having the same preferences over workers as the original firm. Further, the preferences of the workers between replicas of two different firms should be exactly the same as her preferences between the originals. On the other hand, the non-cooperative theory where each firm employs more than one worker is considerably different from the non-cooperative theory where firms may employ at most one. It is noteworthy that the cooperative theory for many-to-many two-sided matching models does not permit the same replication argument. This has been shown in Lahiri (2006).

The analysis reported in this paper, attempts at extending results pertaining to the existence of stable matchings in a labor market, by introducing technology as an essential determinant of the results that we obtain. Since our paper, is concerned with the cooperative theory of three-sided systems, the model that we use of a firm employing at most one worker, continues to provide valuable insights concerning the existence of stable matchings in labor markets.

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Parameter Estimation by the Unequal Intervals

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Keywords: *Statistical game, Zero-sum game, Parameter estimation*

Abstract: *A Hide and Seek problem reduced to a statistical game. In this game Statistician estimates unknown parameter by m different intervals. Payoff of Statistician is a number of intervals that cover the parameter. An optimal equalized strategy of Statistician and game value are found.*

One Step Seek games are well known in game theory. Such games appear in planning of military operations (detection of submarine by various detectors) and in planning of purchase at high uncertainty market. In these zero-sum games a payoff function is a step function or has only two values in simple situations. These properties of payoff function help us to construct a game solution in a closed form even if common existence theorems are not applied.

In this article we consider a game $\Gamma(\mathbf{a}) = < I^m, I, k >$, where the second player (Infiltrator) hides an object in a point *y* of unit segment; the first player (Guard) chooses a set of *m* intervals with lengths $a_1, a_2, \ldots, a_m$ and the payoff function *k* is

$$k(\mathbf{x}, y) = \sum_{i=1}^{m} \mathbf{1}_{[x_i, x_i + a_i)}(y) = \sum_{i=1}^{m} \mathbf{1}_{[0, a_i)}(y - x_i)\ ,\ \ y \in I = [0,1]\ ,\ \ \mathbf{x} = (x_1, x_1, \ldots, x_m) \in I^m\ .$$

Thus the payoff equals a number of intervals that covers the point *y*. Special case of game $\Gamma(\mathbf{a})$ is considered by Baston V.J., Bostok F.A., Ferguson T.S. (1989). The game $\Gamma(\mathbf{a}) = < I^m, I, k >$ is similar to game $\Gamma_{\max} = < I^m, I, k_{\max} >$ with payoff function

$$k_{\max}(\mathbf{x}, y) = \max_{1 \le i \le m} \left(\mathbf{1}_{[x_i, x_i + a_i)}(y) \right)$$

The last game is considered by Ruckle, (1983) and Garnaev, (2000) and interpreted as an Infiltration Game with *m* Cables.

On the other hand the game $\Gamma(\mathbf{a})$ is a partial case of a statistical game $\Gamma = < X^m, Y, k >$ with payoff function

$$k(\mathbf{x}, y) = \sum_{i=1}^{m} k_i(x_i, y) \text{ , where } \mathbf{x} = (x_1, x_2, \ldots, x_m) \text{ , } x_i \in X \text{ , } i = \overline{1,m} \text{ .}$$

If $m = n+1$ and $p_i(y) = C_n^{i-1} y^{i-1}(1-y)^{n-i+1}$ we have a statistical game with payoff function

$$k(\mathbf{x}, y) = \sum_{i=1}^{n+1} p_i(y) w_i(x_i, y)$$

for estimation problem of a parameter y of the binomial distribution $B(n, y)$. The game solution can be found in a book of Blackwell, Girshick, (1954) if $w_i(x, y) = \varphi(y)(x-y)^2$. If the functions $w_i(x, y) = 1_{[0,a)}(y-x)$ then we have statistical game for interval estimation problem. And the game value is confidence probability. A solution of the game is found by Lutsenko (1990), Lutsenko, Maloshevskii, (2003).

Let $\mathbf{a} = (a_1, a_2, \ldots, a_m)$ be a set of positive numbers and $\mathbf{h} = (h_1, h_2, \ldots, h_m)$ be a set of nonnegative integers. We construct a base set of discrete measures: $\bar{\sigma}(\mathbf{a}, \mathbf{h}) = (\sigma_1, \sigma_2, \ldots, \sigma_m)$.

At first for the sequence $h_1, h_2, \ldots, h_m$ we compute partial sums:

$$L_i : \ L_0 = 0 \text{ , } L_i = \sum_{i=1}^{i} h_s \text{ , } i = \overline{1,m} \text{ .}$$

Now we can define a sequence of m points: $c_1 = 0$, $c_l = c_{l-1} + a_i$, if $l \in [L_{i-1}+1, L_i]$ and then discrete measures

$$\sigma_i = \sum_{l=L_{i-1}+1}^{L_i} \delta[c_l] \text{ .}$$

If the segment $[L_{i-1}+1, L_i]$ is empty then we put: $\sigma_i = 0$.

The set of measures $\bar{\sigma}(\mathbf{a}, \mathbf{h}) = (\sigma_1, \sigma_2, \ldots, \sigma_n)$ declares the next actions of the first player. At first he lays h_1 intervals with length a_1, from the left part of unit segment.

Then from the right he joins h_2 intervals with length a_2, afterwards joins h_3 intervals with length a_3 and so on.

Theorem 1. For any positive vector $\mathbf{a} = (a_1, a_2, \ldots, a_m)$ the game $\Gamma(\mathbf{a})$ has a solution, the first player has an equalizing strategy that is a linear combination of base set of discrete measures: $\bar{\sigma}(\mathbf{a}, \mathbf{h}) = (\sigma_1, \sigma_2, \ldots, \sigma_m)$.

Theorem 2. If the matrix of coefficient of linear programming problem

$$F(\mathbf{p}) = \sum_{j=1}^{n} p_j \to \max \ , \tag{1}$$

$$\begin{cases} \sum_{j=1}^{n} h_{i,j} p_j = 1, \ i = \overline{1,m}; \\ \quad p_j \geq 0, \ j = \overline{1,n}. \end{cases} \tag{2}$$

consists of nonnegative integers and the problem has a solution, vector $\mathbf{p}^* = (p_1^*, p_2^*, \ldots, p_n^*)$ is an optimal plan of the problem and the dual problem for problem (1, 2)

$$G(\mathbf{q}) = \sum_{i=1}^{m} q_i \to \min$$

$$\sum_{i=1}^{m} h_{i,j} q_i \geq 1, \ i = \overline{1,n};$$

has a solution and vector $\mathbf{q}^* = (q_1^*, q_2^*, \ldots, q_m^*)$, $q_i^* > 0 \quad i = \overline{1,m}$ is an optimal plan of the problem and

$$q_1^* = \alpha_1 / N \ , q_2^* = \alpha_2 / N \ , \ldots , q_m^* = \alpha_m / N \ , \alpha_1, \alpha_2, \ldots, \alpha_m \in \mathbf{N}$$

then for any vector $\mathbf{a} = (a_1, a_2, \ldots, a_m)$ that coordinate lays in intervals $a_i \in \left(\frac{\alpha_i}{N}, \frac{\alpha_i}{N-1} \right]$, $i = \overline{1,m}$, the value of game $\Gamma(\mathbf{a})$ equals

$$v(\Gamma) = p_1^* + p_2^* + \cdots + p_n^* = q_1^* + q_2^* + \cdots + q_m^* \ ,$$

optimal strategy of the first player is the linear combination of measures from base set of mesures

$$\mu^* = (\mu_1, \mu_2, \ldots, \mu_m) = \sum_{j=1}^{n} p_j^* \cdot \bar{\sigma}(\mathbf{a}, \mathbf{h}_j) \quad \mathbf{h}_j = (h_{1,j}, h_{2,j}, \ldots, h_{m,j}) \ , \ j = \overline{1,n} \ ,$$

optimal strategy of the second player is the discrete measure

$$\nu^* = \frac{1}{N} \sum_{i=1}^{N} \delta[y_i] \ , \ y_i = \frac{i-1}{N-1} \ .$$

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A Competitive Dynamical Linear Model of Economy

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Keywords: *Competitive model, Economy, Equilibrium*

A linear competitive multi-step model Γ of economy with $|N|$ players (agents $N_1, N_2, ..., N_{|N|}$) is considered in the paper. There are technological processes $m(i = 1...m)$ in the linear economy Γ, which is described by a technological matrix A_Γ consisting of elements a_{ij}.

Here the number a_{ij} is a quantity of the product G_j (produced, if $a_{ij} > 0$, and consumed if $a_{ij} < 0$) when the technological processes P_i functionates with intensity equal to one.

$$A_\Gamma = \begin{array}{c} \\ P_1 \\ \vdots \\ P_m \end{array} \begin{array}{c} \begin{array}{ccc} G_1 & \cdots & G_n \end{array} \\ \begin{pmatrix} a_{11} & \cdots & a_{1n} \\ \vdots & \ddots & \vdots \\ a_{m1} & \cdots & a_{mn} \end{pmatrix} \end{array}$$

So the state of the economy Γ is defined by a point A_Γ in the Euclid space E_{mn} of dimension $m \times n$. It is supposed that the economy Γ evolutionates in the Euclid space E_{mn} in accordance with a difference equation

$$A_t = F^t(A_{t-1}, u_1, u_2, ..., u_{|N|}), u_k \in U_k \subset E^{m_k}, k = \overline{1, |N|}, t = \overline{0, T}$$

Here u_k are control parameters of the agents $N_k, k = \overline{1, |N|}$. In the simple linear case the difference equation of the economy Γ evolution is described in the following way.

$$A_t = F^t A_{t-1} + B_1^t u_1 + B_2^t u_2 + ... + B_{|N|}^t u_{|N|}$$

The linear dynamical competitive process of the economical model Γ evolution under consideration starts at an initial moment $t=0$ from an initial state A_0 and terminates at the moment T at a final point A_T. At the final point A_T the agents $N_k, k=\overline{1,|N|}$ get their payoffs (profits) $H_i(A_T)$ described by means of continuous profit functions defined over the state space E_{mn}. For the case of finite sets of agents control parameters algorithms are given to find equilibrium points and compromise solutions. Two numerical examples are solved.

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Solutions of Bimatrix Coalitional Games

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Keywords: *Games, Bimatrix coalitional games*

The new approach to the solution of bimatrix coalitional games is proposed. Suppose N person game Γ with finite sets of strategies is given. The set of players N is divided on two subsets (coalitions) S, N\S each acting as one player. The payoff of player S (N\S) is equal to the sum of payoffs of players from S (N\S). The Nash equilibrium (NE) in mixed strategies is calculated (in the case of multiple NE [1] the solution of the correspondly coalitional game will be not unique). Then the payoff of coalitions S, N\S which appears with positive probability in the NE are allocated according to the Shapley [2] value. After the mathematical expectation of the Shapley value components is taken with respect to the probability distribution over the 2-tuples of strategies generated by NE. The resulting payoff vector will be a generalization of the PMS-value defined and computed in [3] for coalitional games with perfect information.

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A Game Model of Economic Behavior in an Institutional Environment[1]

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Keywords: *Game, Government, Institutions, Myopic agents, Nash equilibrium*

Galbraith (1983) defined *power* as “the ability of individuals or groups to win the submission of others to their purpose”. Political and economic *institutions* often serve to force economic agents to act in a definite way. As North (1990) noted, “In the jargon of economists, institutions define and limit the set of choices of individuals’.

In the present paper a game model is proposed helping to reveal a relation between institutions and decisions of private agents. Players in the game are a *government* and numerous *private agents*. Activities of the private agents are modeled as paths on a given oriented graph with a finite set of nodes M (states) and a set of arcs N. The government (the first player) establishes and announces an institutional system P, and after that the private agents choose their paths.

A move of a private agent from the node i to the node j yields her an instant utility gain $u(i,j)$. Agents receive similar utilities if they choose the same activities. However, their choice depends not on their preferences only but on their initial states as well. Each private agent solves the following problem:

$$\max \sum_{t=0}^{T} \beta^t (u(i_t, i_{t+1}, P)) ,$$

where $(i_t, i_{t+1}) \in N$, $0 \le \beta \le 1$ is a discount factor, i_0 is an initial state of the agent, and P is an institutional system established by the government. An agent is *myopic* if she

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possesses a zero discount factor $(\beta = 0)$ or a zero horizon ($T = 0$) (These two cases are close though not identical). An optimal path for a myopic agent is found stepwise.

An *institutional system* is a mapping $P: N \to A$ where A is a set of possible actions of the government (e.g. values of taxes and subsidies, measures of punishment, etc.) in response to the agents' moves. Each move (i, j) of a private agent yields an instant utility $v(i, j, P)$ to the government. The government wishes to maximize a discounted sum of utilities yielded by all private agents given the government's discount factor, $0 \le \beta_g \le 1$. The government possesses information on the agents' preferences but it has not enough information about their number (e.g. not all agents are active) and their initial states.

Under this partial information, is it possible for the government to establish a consistent institutional system corresponding to a Nash equilibrium? The paper provides the following answer: the government is able to do this if the private agents are myopic. A constructive procedure solving this problem is proposed based on methods of 'extremal algebra' (with operations $a \otimes b = a + \beta b, a \oplus b == \max\{a,b\}$. See Matveenko, 1990, 1998 for applications of extremal algebra to schemes of dynamic programming without and with discounting).

A possible application of the game (see Kraynov, Matveenko, 2007a, 2007b) is a problem of effectiveness of the government control in science and R&D sector in Russia which became actual in connection with a reform started by the government recently. Here *M* can be interpreted as a set of possible themes which can be explored by researchers, and *N* contains directions of new research. The government will be more satisfied with themes related to some definite practical issues. The model shows that if the researchers are myopic (i.e. are interested more in their current achievements and welfare than in long-term perspectives of their activities) then the government is really able to establish effective incentives under incomplete information. However, if the researchers are long-term-oriented, the government's problem is insolvable.

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Fish Wars with Changing Area for Fishery[1]

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Keywords: *Cournot-Nash equilibrium, Bioresource management problem, Dynamic games, Incentive equilibrium*

Abstract: *In this work we use the approach, where the policy of the center is to determine the optimal prohibited share of the aquatic environment, for bioresource sharing problem with two players.*
We consider here a discrete-time game model related with the bioresource management problem (fish catching). The center (referee) shares a reservoir between the competitors. The players (countries), which harvest the fish stock are the participants of this game.
We consider three types of the biological growth rule for fish population. We suppose that the utility function of country i is logarithmic and players wish to maximize the sum of discounted utility functions.
We derive the infinite horizon Cournot-Nash policies by finding finite horizon Cournot-Nash policies and letting the horizon tend to infinity [3]. Also we determine cooperative solution and optimal steady-state size of the population. We consider the center as a player and find Stackalberg equilibrium for different functionals determining the center's gain. Also we investigate the cooperative incentive equilibrium [2] in the case when the center punishes players for a deviation. The numerical modelling and the results comparison are given.

Discrete-time model

Let us divide the water area into two parts: s and $1-s$, where two countries exploit the fish stock. The center (referee) shares the reservoir. The players (countries) which exploit the fish stock on their territory are the participants of this game.

The fish population grows according to the biological rule:

$x_{t+1} = (\varepsilon x_t)^{\alpha}$, $0 < \alpha < 1$.(1)

We suppose that the utility function of country i is logarithmic:

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$$I_1 = \ln((1-s)x_t u_t^1),\ I_2 = \ln(s x_t u_t^2),$$

where $u_t^1, u_t^2 \geq 0$ – countries' fishing efforts at time t.

We consider the players' net revenue over a fixed time period $[0,n]$:

$$J_1 = \sum_{t=0}^{n} \beta_1^t \ln((1-s)x_t u_t^1),\ J_2 = \sum_{t=0}^{n} \beta_2^t \ln(s x_t u_t^2), (2)$$

and with infinite horizon

$$J_1 = \sum_{t=0}^{\infty} \beta_1^t \ln((1-s)x_t u_t^1),\ J_2 = \sum_{t=0}^{\infty} \beta_2^t \ln(s x_t u_t^2), (3)$$

where $0 < \beta_i < 1$ – the discount factor for country i.

We find the Cournot-Nash and cooperative equilibria for the problem (1)-(2). We receive the infinite horizon Cournot-Nash and cooperative equilibria for the problem (1)-(3) and determine the steady-state solution.

Statement 1. *The Cournot-Nash equilibrium of the problem (1)-(3) are*

$$\bar{u}_1 = \frac{\varepsilon\beta_2(1-\alpha\beta_1)}{(\beta_1+\beta_2-\alpha\beta_1\beta_2)(1-s)},\ \bar{u}_2 = \frac{\varepsilon\beta_1(1-\alpha\beta_2)}{(\beta_1+\beta_2-\alpha\beta_1\beta_2)s}.$$

The steady-state size of the population under Cournot-Nash policies is

$$\bar{x} = \left(\frac{\varepsilon\alpha\beta_1\beta_2}{\beta_1+\beta_2-\alpha\beta_1\beta_2}\right)^{\frac{\alpha}{1-\alpha}}.$$

We consider the cooperative incentive equilibrium and assume that the center punishes the players for a deviation from the equilibrium point, but not themselves, as it was in [1]. The cooperative incentive equilibrium is also determined for finite and infinite planning horizon.

Statement 2. *The incentive equilibrium of the problem (1)-(3) is*

$$\gamma_1(u_2) = \frac{\varepsilon\mu_1(1-\alpha\rho)}{1-s_2^*},\quad \gamma_2(u_1) = \frac{\varepsilon\mu_2(1-\alpha\rho)}{s_1^*},$$

where

$$s_2^* = s^d - \frac{s^d}{u_2^d}(u_2 - u_2^d),\ s_1^* = s^d + \frac{1-s^d}{u_1^d}(u_1 - u_1^d),$$

and u_1^d, u_2^d is the cooperative equilibrium of the problem (1)-(3).

Also we investigate the models with the biological growth rule for fish population of the forms:

$$x_{t+1} = (\varepsilon s x_t)^{\alpha},\ 0 < \alpha < 1$$

and

$$x_{t+1} = (\varepsilon x_t)^{\alpha s},\ 0 < \alpha < 1$$

In these cases we suppose that one of the players follows a renewal policy.

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Situations of Absolute Equilibrium by Nash in a Dynamic Game

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Keywords: *Equilibrium, Strategy of behavior*

Abstract: *In this work situations of absolute equilibrium by Nash are introduced and explored for a non-antagonistic dynamic game. Here the set of alternative choices of players to realize strategies is unlimited. The strategies of behavior for the considered game are defined.*
There is a well-known theorem about existence of the situation of absolute equilibrium by Nash in the finite game with perfect information in the pure strategies, and the structure of the set of situations of absolute equilibrium by Nash is explored.
An algorithm for construction of situations of absolute equilibrium by Nash is found in this game with perfect information in the class of strategies of behavior. It's shown that every situation of absolute equilibrium in the considered game may be received by this algorithm.

Let's consider a non-antagonistic game, dynamics of which is described by the differential equation

$$\dot{x} = f\left(x,t,u_1,...,u_n\right), \tag{1}$$

where f is from C^1, $x \in R^m$, and $u_i(t)$, $i = 1,...,n = I$ are functions from the class of measurable functions. $n \in N$ players participate in the game. The moments t_0 – start (beginning) of the game, ϑ – end of the game are given and also some finite division of the segment of time $[t_0, \vartheta]$ is given by means of points $t_0 < t_1 < ... < t_l < \vartheta$, $l \in N$. The point of division t_α, $\alpha \in \{1,...,l\} = J$, is the moment of time when, according to some order, one of the players $i \in I$ must choose a function $u_i(t) \in \Omega_i$, $t \in [t_\alpha, t_{\alpha+1})$ according to its strategy, where Ω_i, $i \in I$ is a compact restrain set in R^{m_i}, $m_i \in N$.

Let's denote a collection of functions by $u \mid u_i = \left(u_1,...,u_{i-1},u_{i+1},...,u_n\right)$. If the player $i \in I$ hasn't done a step till some moment of time t_α, then in (1) as a control function of that player in the argument of f can be put each of $u_i(t) \in \Omega_i$.

The game starts from the position $x_0 \in R^m$ at the moment t_0. Let's put the following restrictions on the right parts of equation (1)

a) there exist a uniform bound for all responses $|x(t)| < b$ on $t_0 \le t \le \vartheta$

b) for each fixed $(x,t,u \mid u_i)$, $i \in I$, the set $V(x,t) = \{f(x,t,u_1,...,u_n), u_i \in \Omega_i\}$ is convex, and as it's follows from [1] the set $V(x,t)$ is compact and convex.

Definition 1. For each $i \in I$, $U \mid U_i$, (t_0,x_0) the set of solutions of the system (1) $x(t,x_0,t_0,U \mid U_i)$ corresponding to admissible functions $u_i(t)$ are defined on $[t_0,t]$, $t_0 \le t \le \vartheta$. The set of endpoints of solutions $x(t,x_0,t_0,U \mid U_i)$ is called a set of attainability of the system (1) on $[t_0,t]$. We'll denote it $K(x_0,t)$ or $K_i(x_0,t)$ if it is necessary to indicate the player.

From the conditions put on f it follows that the sets $K(x_0,t)$ are compact and varies continuously by t for $t_0 \le t \le \vartheta$.

A bunch of solutions $x(t,x(t_\alpha),t_\alpha)$ of the system (1) comes out from each point $x(t_\alpha) \in K(x_0,t_\alpha)$, each corresponding to different choices of $u_i(t) \in \Omega_i$ of the player i, $t \in [t_\alpha,t_{\alpha+1})$. To each definite choice $u_i(t)$ of the player i corresponds a unique solution of the system (1) $x(t,x(t_\alpha),t_\alpha)$. At the moment $t_{\alpha+1}$ the player that does the step, proceeds the game from the point $(x(t_{\alpha+1}),t_{\alpha+1})$ and keeps the continuity of the solution. We'll denote the set of positions that succeed $(t_\alpha,x(t_\alpha))$ by $Z(x(t_\alpha))$. This is the set of attainability $K_i(x(t_\alpha),t_{\alpha+1})$.

Definition 2. The finite treelike oriented graph T with the root x_0 is called the tree of the game.

Let's denote by $T(x)$ the subtree of the tree T with the beginning at the position x. Let points $A_1,...,A_n$ are given in R^m and also positive numbers $e_1,...,e_n$. The aim set M_i of the player $i \in I$ we'll define by the condition: $M_i = \{x \in R^m : \rho(x,A_i) \le e_i\}$, $i \in I$, where $\rho(a,b)$ is the distance between points a and b, $a,b \in R^m$, and e_i are positive numbers.

The tree of the game $K(x_0)$ with the structure given below we'll call the positional game $\Gamma(x_0)$. All positions in the game are divided into $n+1$ compact sets in R^m $P_1, P_2, ..., P_n, P_{n+1}$, that is called division into the sets of turn. Positions that belong to the set P_i are positions of the player i, from where it does its choice of function u_i. The positions from P_{n+1} are final. Numbers $h_i\left(x(\vartheta, x_0, t_0)\right) = \rho\left(x(\vartheta, x_0, t_0), M_i\right)$, $i \in I$ that are called payoffs of players are given in the final positions.

The goal of each player is to achieve the minimum of distance of the point $x(\vartheta, x_0, t_0)$ from its aim set, i.e. to minimize the function $h_i\left(x(\vartheta, x_0, t_0)\right)$ by means of choosing its controlling functions.

Definition 3. One-valued reflexion $b_i(\cdot)$, which, for each position $x \in P_i$ puts in correspondence some probability distribution $P_x = \{P(y), y \in Z(x)\}$, $\int\limits_{Z(x)} P(y)dy = 1$ on the set of alternatives $Z(x)$, or the function of distribution $F(y)$, $y \in Z(x)$ so that $\int\limits_{Z(x)} dF(y) = 1$, is called the strategy of behavior of the player i. Here the integral is considered the integral of Lebeg.

Let's denote the set of all possible strategies of behavior of the player i by B_i.

In the game $\Gamma(x_0)$, each set $\left(b_1(\cdot), ..., b_n(\cdot)\right)$ of strategies of behavior we'll call situation.

There is a well-known theorem about existence of the situation of absolute equilibrium by Nash in the finite game with perfect information in the pure strategies [2], [4] and in [3] the structure of set of situations of absolute equilibrium by Nash is explored.

In this work situations of absolute equilibrium by Nash are introduced and explored for a dynamic game. Here the set of alternative choices of players to realize strategies is unlimited.

An algorithm for construction of situations of absolute equilibrium by Nash is found in this game with perfect information in the class of strategies of behavior. It's shown that every situation of absolute equilibrium in the game $\Gamma(x_0)$ may be received by this algorithm.

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Models of Optimal Organizational Hierarchies[1]

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Keywords: *Discrete optimization, Management hierarchy optimization, Organizational structure formation*

Abstract: *We review the recent literature on mathematical models of management hierarchy formation and propose a new normative economic model of hierarchy optimization. An optimal hierarchy is supposed to minimize the running costs of a firm. These costs include direct maintenance expenses along with wastes from a loss of control. The problem of optimal hierarchy is solved analytically. We derive formulas for the span of control, headcount, efforts distribution, wages differential, etc, as functions of exogenous parameters. Prospective studies may be devoted to the model identification from empirical data and business administration literature.*

We combine the transaction costs approach with the original mathematical results in an optimal hierarchy design (Mishin S.P., 2004; Goubko M.V., 2006) to formulate and investigate the models of multi-layer management hierarchies. Our approach assumes reducing the problem of rational management hierarchy to the model of discrete optimization (to choose a hierarchy from a finite set of feasible hierarchies with the aim to maximize a certain criterion that maps the set of feasible hierarchies into the number scale).

Formal models of intra-firm hierarchies are studied by economists since the 60s of XX century. The main issue addressed by the earlier papers (see M. Beckmann (1960), O.E. Williamson (1967), G.A. Calvo and S. Wellisz (1978)) is whether the loss of control in a management hierarchy limits the growth of a firm. The later literature pays more attention to the explanation of empirical data on executives' compensation in big firms (S. Rosen (1982), V. Smeets and F. Warzynski (2006)). These issues are important for the theory of the firm but business administration still seeks for formal normative models offering a clue to the formation of competitive management hierarchy.

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We propose a normative model of management hierarchy optimization. The questions posed by this model are: how many managers the firm must hire, when headcount should be increased or decreased, how managers wages depend on their positions, whether corporate information systems implementation results in a flatter management hierarchy, when the growth of the firm is advantageous, etc.

We study a manufacturing firm that chooses one of available products to produce. The product determines the technology and, thus, the set of productive workers. The hierarchy of managers built over the set of workers provides monitoring and coordination. Two central points of management hierarchy are considered – the maintenance costs and the loss of control in a chain of command.

The maintenance costs of a manager depend on her span of control, position (size of a unit under control), monitoring efforts, and the firm's production plan. The costs also depend on two external parameters – the level of managers' ability and the degree of standardization of business processes in the firm. The wastes from the loss of control increase exponentially with a hierarchy level.

The problem in hand is the problem of the principal – to choose a product, production plan, and to organize the efficient execution of this plan, i.e. to find out how many managers to hire, how to subordinate both workers to managers and managers to higher-layer managers in order to obtain better efforts (and thus, the output) at lower costs.

We consider a framework with the manager's cost function obeying constant elasticity with respect to the size of a unit under control. For such setting the technique has been developed earlier (Goubko, 2006) to solve the optimal hierarchy problem. We prove the optimal management hierarchy to be uniform (i.e. every manager in a hierarchy has the same span of control) and symmetric (i.e. every manager seeks to divide the subordinate group of workers equally among his immediate subordinates). Then the formulas are obtained for the optimal span of control, the managers' headcount, their compensation amounts, and a production plan. Thus, for the current setting the problem of optimal hierarchy analysis and synthesis is completely solved.

The results allow analyzing the impact of external parameters (such as managers' ability and the degree of standardization) on a firm's size, its financial results, employees' wages and the shape of the optimal hierarchy. Prospective studies may be devoted to the model identification from empirical data and business administration literature.

The mathematical technique developed can be used to solve optimal hierarchy problems in various areas: design of data collection and quality management systems, assembly planning, etc.

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A Stochastic Game Analysis for Financial Securities

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Keywords: *Stochastic game, Mathematical finance, Markov process*

We need sometimes a mathematical financial model that there are **NOT** so many investors for a security. At this situation, it's not appropriate to use Ito process ($dX(t) = u(t,\omega)dt + v(t,\omega)dW(t)$) as a stochastic process for securities, derivatives and so on. We propose a combined method of co-operative game, non co-operative game and stochastic process. We've found that not so many investors (or only a few substantially) for some Japanese companies. Our method is useful for all companies in similar conditions. This method mainly consists of the following analysis.

Coalition Formation Analysis by Co-operative Game

Even if many investors N exist for a security, we often could find that subset of investors form coalitions $\{C_k : k = 1,\ldots,m\}$. If we suppose that $N = \bigoplus_k C_k$ i.e. no one deviates from her/his coalition to check characteristic function v of this game, we would regard each coalition as a player in terms of non co-operative game theory. After this analysis, we replace $N = \{1,\ldots,m\}$ for later non co-operative game analysis.

Find Hidden Markov Points

When each player $i \in N$ takes an action (or a strategy), we suppose that it is an "approximatel" hidden Markov point. It would be better that some actions might be ignored. How to define these points $(t_1,\ldots,t_p)$ precisely depends on economic or other situations.

Stochastic Game -Non Co-operative Game- Analysis

After we found discrete time points in the above way, if we define finite states S, set of actions $A(i)$ for each player $i \in N$, reward functions $r(s,a)$ where $s \in S$ and $a \in \prod_i A(i)$ and transition probability $P(s_{t+1} \mid s_t, a)$. "Actions" are used in terms of

Markov Decision Process. If an optimal policy (strategy) exists and could be calculated in polynomial time, it would be useful information. We have a plan to tell about conditions for its existence in my talk. Also, we may ignore actions for simple analysis.

A rough outline of our method as follows:

- Step1: Define investors=players for a security.
- Step2: Coalition Formation Analysis. Redefine players for non co-operative game analysis.
- Step3: Define non co-operative game (or Stochastic Game) Define pure strategies -actions-, reward -payoff- functions and transition probabilities.
- Step4: Find hidden Markov points.
- Step5: Calculate equilibriums for each hidden Markov point.
- Step6: Define states from results of equilibrium and other info.
- Step7: If the number of states is so many, re-define states and reduce the num of them.
- Step8: Estimate transition probabilities.
- Step9: Check time-homogeneous. If NO go to Step3.
- Step10: Check Markov property. If NO go to Step 6 or Step 7
- Step11: Calculate characteristics
- Step12: Error Analysis. If results are rejected, check the following Steps, otherwise STOP.

  - If you find large error in Markov property check, go to Step 10.
  - If you find large estimation error in transition probability, go to Step 7.
  - If you find other problems, go to Step3.

Research on the Problem of Evaluating Cooperative Games; the Method of Agencies and the Search for Confirmation through Model Variants

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Actually, although this is probably not widely known, it was Von Neumann himself who first gave "evaluations" for some zero-sum cooperative games of three players. (In that special case his quite natural evaluation happened to coincide with what would be obtainable by using either the "Shapley value" or the "nucleolus").

But later, when the book of Von Neumann and Morgenstern was published, this idea did not appear in the book and the general category of games of three or more players was studied by an approach that did not give "evaluations" but which did give the basis for subsequently valuable concepts like that of "the core" of a game.

Nash studied two person cooperative games around 1950 and arrived at a broadly applicable theory for the specific case of "Two-Person Cooperative Games" (which became the title of a paper in Econometrica).

And now I have been working on an approach to the extension of cooperative games theory to more than two players.

This is an area in which a few other students have been also working and some interesting ideas have appeared.

For example, the method of "random proposers" operates to reduce a 3-parties game to an averaging of results obtained by studying three related games that can be considered as non-cooperative games and which thus can be studied in terms of existing theory and methods.

In connection with these approaches Armando Gomes, in particular, has naturally derived a "CBV" concept that has interesting connections with the Shapley value and the nucleolus.

My own approach, involving the "method of agencies" was inspired by the observation of how, in biologically oriented studies, it had been found that cooperation would quite naturally EVOLVE in the context of a repeated game even if the game, at each instance of the repetition, was as unfavorable for cooperation as the "Prisoners' Dilemma" game.

The method of agencies involves cooperation being achieved by one player (electively) ACCEPTING the leadership of another player, or in another description the accepting player would be authorizing the accepted player to act as an agent representing the accepting player.

We tried this out first for two-person cooperative games essentially of the type of games of bargaining but with a curved Pareto boundary of the set of attainable modes of cooperation. It worked well, one player would accept the other's leadership or vice versus and effective and efficient cooperation was realized.

Then the study moved into three-person games of a type simply defined by a characteristic function (of the sort of Von Neumann and Morgenstern). For 3-person games two steps of simple acceptance are necessary to move to full cooperation, where the grand coalition (of all three players) would be represented by one of them who had become the agent elected to represent the interest of all players.

And this study of a model for cooperation at the level of three players became a matter of using advanced computer methods for the calculation of the equilibria (of a repeated game model) and the computer software of "Mathematica" was used to make this feasible.

Also, the complexity and extensiveness of the computations that naturally were needed seemed to justify assistance in the work and a project with the NSF (National Science Foundation) became the supporter of the work of a sequence of student assistants. Their work was involved in the calculations for the points on some charts that I can display.

Subsequently to the development of the first model studied through extensive calculations there have been some related experiments that were carried out in Cologne, Germany and also I have been thinking about variations of the repeated game modeling that would use "attorney agents" who would not be themselves drawn from among the original players of the game. However, I have found met some difficulties in this area of study so that progress has not been rapid.

So it is not yet achieved that the results of the extensive computational study of a first model have been confirmed by similar results from the study of a model with “attorney agents”.

Journals in Game Theory

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Existence Conditions for Kernels Generated by Families of Coalitions[1]

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Keywords: *Cooperative games, Kernel*

The theory of the bargaining set and the kernel for cooperative TU-games develops more than forty years. The existence theorems for the kernel and the bargaining set M_1^i are well known (see [3], [4], [9], [5]).

Naumova [6] generalized the bargaining set M_1^i. For each family of coalitions A, she permited objections and counter-objections only between the members of this family. The resulting A-bargaining set is denoted M_A^i. In this spirit a A – kernel, K_A, which is contained in M_A^i, was also defined.

Later she defined [8] another generalization of M_1^i, denoted by M_A^i and the corresponding generalization of the kernel K_A, where $K_A \subset M_A^i \subset M_A^i$, but neither $K_A \subset K_A$ nor $K_A \subset K_A$.

If A is the set of all singletons, then $M_A^i = M_A^i = M_1^i$ and both K_A and K_A coincide with the kernel.

The modifications differ in the definition of counter-objections. In the first version it was possible to use for counter-objections of L against K coalitions Q such that $L \subset Q$, $K \not\subset Q$, and in the second version $L \subset Q$, $K \cap Q = \emptyset$. For generalized kernels, K_A considers excesses of coalitions which contain objecting coalition and does

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not contain the coalition under objection, and K_A considers excesses of coalitions which contain objecting coalition and does not intersect the coalition under objection.

Necessary and sufficient condition on A which guarantee that each TU-game would have a nonempty K_A was described in [7]. Now it is proved that this condition is sufficient for existence of nonempty K_A for NTU-games (for a special generalization of K_A to NTU-games).

Sufficient conditions and necessary conditions on A for existence of K_A for TU-games and sufficient conditions on A for existence of K_A for NTU-games are obtained.

All these sufficient conditions are formulated in terms of special directed graphs, where A is the set of vertices. For fixed A, they can be verified in a finite number of steps.

The new existence theorems are based on generalizations of the results of Peleg in [10] and Billera in [2].

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Competition of Large-scale Projects: a Game-theoretical Model with Exponential Functions

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Keywords: *N-player non-cooperative game, Nash equilibrium solutions, Competition of large-scale projects*

Mathematical and computer models of the process of competition of investors to large-scale projects, such as construction of gas pipelines, was studied in many publications, see e.g. [1]. In the paper [2], a description was obtained of Nash equilibrium points in a game of two investors who finance competing projects of gas pipeline construction. It was shown in [3] that if instead of a complete description, we are satisfied with an algorithm that enumerates all the points of Nash equilibrium, then we can consider a game of several investors and significantly relax the imposed mathematical requirements.

The games considered in the above cited articles provide mathematical models for the following situation. Several gas pipelines are being built by competing investors and are aimed at one and the same regional market of natural gas. When new gas pipelines come into operation, the amount of gas supplied to the market is increased, which obviously can lower the price of gas.

The investors that put their pipelines into operation earlier can enjoy a high price of gas for some time. The investor who comes to the market first enjoys some period of monopoly. On the other hand, completing the construction of a pipeline later can be desirable for a number of reasons. In particular, it can reduce the price of construction. This naturally creates a kind of a game between the investors. Various aspects of this game were studied in the above cited articles.

In particular, the above described research included mathematical and computer modeling of the Turkish gas market. These considerations are based on the assumption that the price of gas is set by the market itself. Some heuristic algorithms were proposed. A rigorous mathematical model was developed and published later in [2]. This model has initiated further development of computer realizable algorithms and mathematical generalizations in many ways.

In the further research, an attempt was made to apply the developed technique to the Chinese market of natural gas. However, many of the assumptions used for modeling the Turkish market appeared to be invalid for the specific Chinese market. From the point of view of economics, the main difference is that in the case of China the prices are fixed not purely by a market mechanism. A mathematical model that takes into account these circumstances was given in [4]. Methodically, the article [4] is a continuation of research [2], where the basic model is described. However, the assumptions of the model of Chinese market are essentially different and sometimes the opposite.

In the paper the results and algorithms of [3] are specified for a typical case when the process of construction and exploitation of the gas pipelines is described with the help of exponential functions [5]. This allows us to simplify many of considerations and imposed conditions. On the other hand, this supposition is not too restrictive as exponential functions frequently arise in connection with problems of this type and in research on economics in general. It is convenient to work with exponential functions as each of them is described by two parameters only. Algorithms in [3] require finding intersection points of the corresponding graphs. In the case of exponential functions, this is reduced to an elementary equation. There is no need to employ numerical methods. Because of that, algorithms from [3] in the case of exponential functions can be effectively realised in the form of computer codes, see [5]. Below we analyze in details the mathematical requirements that arise in these problems in the case of exponential functions, present algorithms for finding best responses of participants and points of Nash equilibrium, and describe the corresponding software.

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Reflexive Games: Modelling of Reflexive Decision-making

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Keywords: *Decision-making, Hierarchy of beliefs, Reflexivity*

Abstract: *The paper contains the survey of the game-theoretical models of reflexive decision-making. In most equilibrium concepts, used in the game theory, the parameters of the game are common knowledge – all agents know it, all agents know that all agents know it and so on ad infinitum. In general case the agents have different beliefs about beliefs of each other, thus an infinite (reflexive) belief structure appears. For this case the concept of informational equilibrium is fruitful. The paper is devoted to the formulation of the reflexive model, and contains conditions of the reflexive equilibrium existence and stability, solution of the reflexivity depth problem for some cases, and examples.*

Nowadays game-theoretical models are widely spread in the descriptions of social and economic systems (e.g. [1-3]). A great variety of social and economic relations generates the variety of games' models. The paper is devoted to the informational aspects of decision-making in conflict situations and, particularly, the role of mutual beliefs of the agents.

Traditionally, in non-cooperative game theory it is assumed that agents choose their actions simultaneously and independently, and information about the game is common knowledge among the agents, i.e. each agent knows: the set of the players, all feasible sets and goal functions; he also knows that all other agents know it, they know, that he knows, etc. ad infinitum. Informally, all agents know the game they play.

To choose the action, maximizing his goal function, the agent has to model (predict) actions of his opponents. The process of such modeling is usually referred to as reflexion [4-6]. At this stage information plays an essential role.

In this paper we consider particular case of agents' beliefs – *point informational structure* (agents have certain beliefs about the value of the state of nature, about the beliefs (also concrete) of other agents, etc.). Consider finite set $N = \{1, 2, \ldots, n\}$ of agents. Let there exists uncertain parameter $\theta \in \Theta$ (the set Θ is common knowledge). Informational structure of the i-th agent I_i includes the following elements. Firstly – the i-th agent's beliefs $\theta_i \in \Theta$ about the state of nature (first-order beliefs). Secondly – his beliefs $\theta_{ij} \in \Theta$ about beliefs of the j-th agent, $j \in N$ (second-order beliefs). Thirdly – his beliefs $\theta_{ijk} \in \Theta$ about beliefs of the j-th agent about beliefs of the k-th agent, $j, k \in N$ (third-order beliefs). And so on. The result is the hierarchy of the i-th agent beliefs.

In other words, informational structure I_i of the i-th agent is defined by the set of all possible values $\theta_{ij_1 \ldots j_l} \in \Theta$, where l runs through the set of non-negative integer numbers, $j_1, \ldots, j_l \in N$.

Analogously, the (point) informational structure I of the whole game is defined by the set of all possible values $\theta_{ij_1 \ldots j_l} \in \Theta$, where l runs through the set of nonnegative integer numbers, $j_1, \ldots, j_l \in N$. Let's stress, that the whole informational structure I is not "observed" by the agents – each of them knows only corresponding substructure.

Thus, an informational structure is an infinite n-tree, with nodes, representing certain information of real or phantom (see below) agents.

Reflexive game is the game, described by the following cortege:

$$\Gamma_I = \{N, (X_i)_{i \in N}, f_i(\cdot)_{i \in N}, I\},$$

where N – set of agents, X_i – set of the i-th agent feasible actions, $f_i(\cdot)$: $\Theta \times X_1 \times \ldots \times X_n \to \Re^1$ – his goal function, $i \in N$, I – informational structure.

It is worth noting, that the term "reflexive game" was introduced by V. Lefebvre in 1965 (see the detailed description in [4]). But the mentioned above paper contains only qualitative discussions of the reflexivity role without any formal models.

Introduce the following notation: Σ_+ – set of all possible sequences of indexes from the set N; $\Sigma = \Sigma_+ \cup \varnothing$; $|\sigma|$ – number of indexes in the sequence σ (equals zero for the empty sequence).

Together with informational structures I_i, $i \in N$, one can consider informational structures I_{ij} (informational structure of the j-th agent from the i-th agent point of view), I_{ijk} and so on. Identifying the informational structure with the agent, described by this structure, one may say that, besides n *real agents* (i-agents, $i \in N$) with informational structures I_i, *phantom agents* (τ-agents, $\tau \in \Sigma_+$, $|\tau| \geq 2$) with informational structures $I_\tau = \{\theta_{\tau\sigma}\}$, $\sigma \in \Sigma$, also take part in the game. Phantom agents exist in the mind of real agents and "influence" on their actions [6-11].

Given the informational structure I, the informational structure of any agent (real and phantom) may be obtained. If it is assumed that any rational agent tries to maximize his goal function, then the process of choosing the action x_τ by the τ-agent is determined by his informational structure I_τ, therefore, knowing this structure, one is able to model his reasoning and find the chosen action. While choosing his action, the rational agent models actions of his opponents (implements reflexivity). Thus, to define the game solution concept (*informational equilibrium*), it is necessary to take into account actions of real agents as well as actions of phantom agents.

One of the main features of Nash equilibrium is self-stability: if the normal form game is repeated several times and all the agents, except i, choose the same components of Nash equilibrium, then the i-th agent has no reason to deviate from the equilibrium. This fact is obviously related to the adequacy of agents' beliefs to reality. In the case of informational equilibrium, the situation may, generally, be different – if in one-shot game some agents (or even all agents) observe the result, different from the expected one. In any case self-stability is destroyed – if the game is repeated again, actions of the agents may alter. But in some cases self-stability may take place even under different (and, moreover, incorrect) beliefs of the agents. Informally, it is possible, when each agent (both real and phantom) observes the expected result of the game. In this case informational equilibrium is *stable*. Stable informational equilibria may be divided into two classes: *true* and *false* equilibria.

Reflexive game is an efficient tool for modeling of interaction of agents, who make their decisions on the base of finite hierarchy of beliefs. If the dependence between the informational equilibrium and informational structure is known, then one can formulate and solve the problems of *informational management* – forming of informational structures, that lead to the required equilibrium.

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Cultural Transmission and the Evolution of Trust and Reciprocity in the Labour Market

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Keywords: *Cultural transmission, Trust and punishment, Social preferences, Cooperation*

In many bilateral relationships where contracts are highly incomplete all the authority is assigned to one player (say the employer). This is known as a hierarchical governance relationship. In some cases the rationale for this asymmetric distribution of authority is the superior information of this player while in other cases it simply reflects different initial wealth endowments. However, this asymmetric distribution of decision rights puts the other side, say the employee, in danger of being exploited, leading to inefficiency if this player refuses to cooperate. Thus, players face a sequential prisoners´ dilemma. This threat of hold up can be mitigated by a preference for reciprocity on the part of the player with authority, by his concern for reputation or finally, by a balance of power, arising from the credible threat by the other player to retaliate if he is exploited. Reputation can only work in a repeated scenario, whereas in this work we focus on a one-shot game. We study the interaction and evolution between the preferences for reciprocity or rewarding of the party with authority and the feasibility to punish hostile behaviour by the player without authority. Notice that it is not enough the willingness to punish, but it is also needed that the rules of the game allow for a sufficient amount of costly punishment. This interaction takes place in an overlapping generations model with heterogeneous preferences and incomplete information, where there is intentioned cultural transmission of preferences. We characterize the long run behaviour of this society, that is, the stable steady states of the dynamics. Our main result states that if the net gains from cooperation are high enough and the amount of feasible punishment is also high the society will converge from any initial condition to an efficient cooperative equilibrium. If any of these conditions does not hold, the society will settle down in an inefficient equilibrium where not all types of players cooperate or although they do it, there is surplus destruction because selfish employers do not reward cooperation. Positive reciprocity (rewarding) is not enough to achieve efficiency. It is also needed a significant allocation of power to the player without authority in order to make the threat of punishment a powerful tool to enhance efficiency and cooperation. Trust and cooperation only operates under the shadow of a credible threat of a significant punishment.

Be the Boss

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The somewhat playful title of this presentation refers to games, both static and dynamic, in which the role of the players is a hierarchical one. Some players behave as leaders, others as followers. Such games are named after Stackelberg. In the current presentation a special type of such games is emphasized, known in the literature as inverse Stackelberg games. In such games the leader announces his strategy as a mapping from the follower's decision space into his own decision space. We will see that such a leader is extremely powerful. Arguments for studying such problems are given. The routine way of analysis, leading to a study of composed functions, is not very fruitful. Other approaches are given, mainly by studying specific examples (one such example, be it an academic one, deals with transaction costs of financial banks). Games with more than one leader and more than one follower will be studied also. As a side issue, expressions like “two captains on a ship”, “the laughing third party” and “divide and conquer” are given a mathematical foundation.

The problem of flow-dependent tolling in optimal toll design will be presented in some detail1 [1]. The road authority, as leader, sets tolls on designated roads such as to minimize the total travelling time of all drivers. The drivers themselves choose their routes such as to minimize their “perceived travel costs”, which is a combination of the actual travel times and the tolls to be paid. Since we assume a large number of drivers, they play according to the – in the traffic community well-known – logit-based stochastic equilibrium concept. As an example, a network of 21 nodes and 58 links will be controlled in this way, both according to the conventional and the inverse Stackelberg equilibrium. The latter case leads to so-called flow-dependent tolling. A counter-intuitive phenomenon that can occur is that the optimal flow-dependent tolling strategy, as

[1] This concerns a PhD study in Delft performed by Katerina Stankova

imposed by the road authority, is a decreasing function the intensity of the flow (i.e. of the traffic density).

Journals in Management

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A Generalized Model of Hierarchically Controlled Dynamical System

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Keywords: *Directed graphs, Dynamical systems, Hierarchical games, Sustainable development*

Abstract: *A generalized model of hierarchically controlled dynamical system is proposed. The model allows to combine several concepts from the game theory and systems analysis. An interpretation of the model for a problem of the sustainable development of an ecological-economic system is given.*

A generalized model of hierarchically controlled dynamic system may be represented as follows:

$$H = \langle N, A, S, C, \{U_i\}_{i \in N}, \{J_i\}_{i \in N} \rangle \tag{1}$$

where $N = \{1,...,n\}$ is a set of players; A is a binary relation of hierarchy on N; S is a set of coalition structures on N, $s \in S$: $s = \{K_1,...,K_m\}$,

$K_1 \bigcup ... \bigcup K_m = N$, $K_i \bigcap K_j = \varnothing$, $K_1 ... K_m \subset N$; C is a controlled dynamical system, $C : x^t = x^{t-1} + f(x^t, U_1^t, ..., U_n^t), x_0 = x^0, t = 1,...,T;$

U_i is a set of strategies of the i-th player; $J_i : U_1 \times U_2 \times ... \times U_n \to R$ is a payoff function of the i-th player.

The following properties are supposed to be fulfilled:

$P1$ – hierarchy: the binary relation A is a strict order relation;

$P2$ – stratification: each coalition structure s ∈ S is ordered (the layers $K_1,...,K_m$ are hierarchically ordered);

$P3$ – economic rationality: each player $i \in N$ tends to maximize J_i

Define a Neumann-Morgenstern characteristic function:

$$v(K) = \max_{u_K \in U_K} \min_{u_{N\setminus K} \in U_{N\setminus K}} \sum_{i \in K} J_i(u_K, u_{N\setminus K}),\ K \subset N\ , \tag{2}$$

where u_K is a set of strategies of the players from K, $u_{N\setminus K}$ is a set of strategies of the players from $N \setminus K$.

The model (1) together with the function (2) allows to combine four known concepts:

1) a directed graph without loops $D = (N, A)$ with additional set of ordered structures of the vertices S, which characterizes a hierarchical structure of the system;

2) a game of n players in normal form $G = \langle N, \{U_i\}_{i\in N}, \{J_i\}_{i\in N} \rangle$ which represents the system as a set of independent rational individuums;

3) a cooperative game $\Gamma_v = \langle N, v \rangle$ which allows to describe coalitions, united actions and rational imputations;

4) a dynamical system controlled by several subjects

$$x^t = x^{t-1} + f(x^t, U_1^t, ..., U_n^t),\ x^0 = x_0,\ t = 1, ..., T\ .$$

The following interpretation of the model (1) for a problem of the sustainable development of an ecological-economic system is possible:

$N = \{L, F\}$, where L (Leader) is a controlling agency, F (Follower) is an enterprise; $A = \{(L, F)\}$ defines the administrative and economic dependence of F from $L;\ S = \{S_1, S_2\},\ S_1 = \{\{L\}, \{F\}\}$ – isolated behavior, $S_2 = \{\{L, F\}\}$ – cooperative behavior of L and F; C is defined as

$$J_L = \sum_{t=1}^{T} [g_L^t(p^t, q^t, u^t, x^t) - M\rho(x^t, X_L^t)] \to \max,\ p^t \in P^t,\ q^t \in Q^t \tag{3}$$

$$J_F = \sum_{t=1}^{T} g_F^t(p^t, u^t, x^t) \to \max,\ u^t \in U(q^t) \tag{4}$$

$$x^t = x^{t-1} + f(x^{t-1}, u^t),\ x^0 = x_0,\ t = 1, ..., T\ , \tag{5}$$

where Q is a set of Leader's administrative strategies; P is a set of Leader's economic strategies; U is a set of Follower's strategies (environmental impacts); J_L is Leader's payoff function considering the sustainability requirement $x^t \in X_L^t,\ M\rho(x^t, X_L^t)$ is a penalty function; J_F is Follower's payoff function; x^t is a state vector of the ecological-economic system in the moment t, x_0 is a known initial state.

Denote $Q = Q^1 \times ... \times Q^T$, $P = P^1 \times ... \times P^T$, $U = U^1 \times ... \times U^T$, then $U_L = \{Q, P\}$, $U_F = U$, J_L, J_F are defined above.

Methods of hierarchical control (compulsion, impulsion, conviction) which permit Leader to provide sustainability $x^t \in X_L^t$, $t = 1,...,T$, are formalized as Stackelberg equlibria of a special form for a hierarchical game of L and F.

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Families of Semipermeable Curves and Their Application to Some Complicated Variants of the Homicidal Chauffeur Problem[1]

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Keywords: *Discontinuous value function, Families of semipermeable curves, Pursuit-evasion game*

Abstract: *For differential games in the plane, the notion of semipermeable curves of the first and second type is introduced. It is demonstrated how one can use such curves to predict discontinuity lines of the value function of time-optimal problems with more complicated variants of the classical homicidal chauffeur dynamics.*

1. Semipermeable curves in the theory of antagonistic differential games are smooth curves in the plane that possess the property: the first player is able to prevent the trajectories of the controlled system from crossing the curve from one (positive) side to the other (negative) side, the second player, conversely, prevents crossing from the negative side to the positive one. The notion of the semipermeability was introduced by Isaacs [1].

By imposing more strict requirements [2] in the definition of semipermeable curves one can achieve that every semipermeable curve will coincide with some motion trajectory of the controlled system for certain "preventing" controls. Using this fact, the first (second) type semipermeable curve can be introduced as a semipermeable curve for which the region adjoining to the positive side of the curve remains to the right (to the

[1] The work was partially supported by RFBR, projects nos. 06-01-00414, 07-01-96085.

left) when moving along the curve with the preventing controls. Properly chosen first and second type curves generate a "road" which can be used by the first player to run the system in the direction towards the fulfilment of his task, whatever the second player does. The semipermeable curves are the boards of such a road. The positive side of the board adjoins to the road. If the first player is unskilful, the system leaves the road, and the return can be only guaranteed after some time which is required to perform a turn and enter the beginning of the road.

The above given verbal description explains the usefulness of the investigation of semipermeable curves in time-optimal differential games in the plane. The value function can be discontinuous in such problems. The discontinuity lines are semipermeable curves of the first and second type.

2. It is important to note that several semipermeable curves of the first (second) type can pass through a given point. There can also be regions where no semipermeable curve goes. Generally, we consider families of semipermeable curves of the first and second type and regions where they are defined. The number of families of the first (second) type is determined by a form of the dependence of the minmax Hamiltonian function on the adjoint variable, namely by a number of roots from minus to plus (plus to minus) of this function at points of the plane. The more families exist, the more complicated (from a game-theoretical point of view) the system dynamics are.

3. The "homicidal chauffeur" game [1,3] is one of the most well-known model problems in the differential game theory. One has two objects: a car (the first player) with a limited turn radius and a constant magnitude of linear velocity, and an inertialess pedestrian (the second player) with a limited speed. Using reference coordinates from [1] one arrives at two-dimensional system controlled by two players. In the case of standard restrictions on the controls of the players, the classical dynamics generate [2] two families of first type semipermeable curves and two families of the second type semipermeable curves. Each family is defined in its own domain. The domains overlap partly.

Having such families (they are only determined by the dynamics and the restrictions on controls) we can find the discontinuity curves of the value function for every particular time-optimal game with classical dynamics. The game is specified through a given target set to which the first player strives to bring the controlled system, whereas the second player strives to prevent this. Let us underline that we can predict the discontinuity curves without solving the time-optimal problem itself, i.e. not having available a precise description of the value function.

4. In [4], a problem with the classical homicidal chauffeur dynamics and modified objectives of the players is investigated. The second player tries to bring the system to a given target set as soon as possible, whereas the first player tries to prevent that. In this case, using the same families of semipermeable curves as for the original problem, we can again predict the discontinuity lines of the value function for this new problem. The difference from the standard case is that now the negative sides of the boards – semipermeable curves – adjoin to the “road”, and it is the second player who ensures the travelling along the road.

5. The structure of families of semipermeable curves becomes essentially complicated if the restriction on the first or second player control depends on the current geometric position. Paper [5] considers an “acoustic” variant of the homicidal chauffeur problem where the constraint of the second player (the evader) depends on the distance of his geometric position from the position of the first player (the pursuer): if the distance between the objects becomes small, the “noise” produced by the evader should be small. In [2], families of semipermeable curves for such dynamics have been investigated. Using these families, a possibility of the explanation to the generation of holes in the capture set where the value function is finite has been demonstrated. The optimal capture time in the holes is infinite.

6. The next complication is connected with the rejection of the constancy requirement on the magnitude of linear velocity of the car. Namely, we will suppose that the magnitude of linear velocity can change instantaneously by taking any values from a given interval. Similar models are used [6, 7] in theoretical robotics. In the talk, a variant of the homicidal chauffeur problem with such a car is considered. Families of semipermeable curves of the first and second type are described. The dependence of the structure and the number of these families on a parameter that specifies the interval for changing the linear velocity of the car is analysed. It is also demonstrated how to predict discontinuity lines of the value function of the corresponding time-optimal differential game using the families of semipermeable curves.

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Multistage Game-Theoretic Model of International Environmental Agreements

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Keywords: *Coalitional game, International environmental agreement, Time-consistent allocation rule, Time-consistency*

The present paper deals with multistage dynamics [1], [2] of international environmental agreements (IEAs) and, in particular, with problem of IEA time-consistency, *e.g.* [3]. Multi-literal collaboration is interpreted as a coalition of heterogeneous players, where target of such cooperation is a stepwise emission reduction over a finite and discrete period of time.

Let $\mathbf{N}$ be a set of players (countries of the world), each of which emits pollutant that damages a shared environment resource. We assume that $\mathbf{N} = \bigcup_{i=1}^{K} \mathbf{N}_i$, $\left(\mathbf{N}_i \cap \mathbf{N}_j = \varnothing,\ i \neq j\right)$, where each subset $\mathbf{N}_i,\ i = 1,\ldots,K,$ consists of N_i *ex ante* symmetric players of type i, having similar payoff functions [4]. Let set S, $S \subseteq \mathbf{N}$ ($S \neq \varnothing$) be a coalition of players, which intend jointly to reduce emission. We consider a two level coalitional game $\Gamma_0(S) = \langle \mathbf{N}, \{q_i^S, q_i^F\}_{i=1}^K, \{\pi_i^S, \pi_i^F\}_{i=1}^K \rangle$, where $q_i^{S(F)}$ are players' strategies and $\pi_i^{S(F)}$ are net benefits of players of type i from S (and $F = \mathbf{N} \setminus S$); the coalition S is the leader and players from set F (free-riders) are the followers.

Assuming a certain agreement with structure S has formed, we determine initial emission reduction targets $(\mathbf{q}^S, \mathbf{q}^F)$ of signatories of the coalition S and free-riders so that they constitute Stackelberg equilibrium in the game $\Gamma_0(S)$. Acting as a leader, signatories choose abatement commitments to be fulfilled by the end of the game by maximizing their aggregate coalitional net benefit. The rest of the players (free-riders) act as followers and give the rational respond (Nash equilibrium) to the abatement decision of coalition members, adjusting their commitments by maximizing individual net benefit.

As soon as optimal abating efforts have been chosen, we go over to the multistage process of stepwise emission reduction during $t = 1,\ldots,m$, assuming that the formed coalition S remains the same during $[0,m)$. Basing on results presented in [5] – [9], we construct a time-consistent abatement scheme, which means that emission reduction $\left(\mathbf{q}^S[t,m), \mathbf{q}^F[t,m)\right)$ during time period $[t,m)$ should constitute Stackelberg equilibrium in the current game $\Gamma_t\left(S, \mathbf{q}^S[0,t-1), \mathbf{q}^F[0,t-1)\right)$, where $(\mathbf{q}^S[0,t-1), \mathbf{q}^F[0,t-1))$ is emission reduction undertaken for the moment t.

Construction of the scheme stipulates that choice of abatement efforts during each following stage is adjusted according to the emission reduction, undertaken during the previous stage, plus taking into account change of the environmental settings. The proposed scheme prescribes large emission reduction to be done during the first stages and then monotonous decrease of abating efforts.

Stability of coalition S is associated with a principle of a self-enforcement (conditions of internal/external stability), introduced in [10] under the formal players' payoffs. These conditions ensure that no player wants to unilaterally deviate, *i.e.* no member of the coalition S prefers to leave the agreement and no non-member from set F prefers to join the coalition. To analyze prospects for sustainable cooperation among players during stepwise emission reduction, we introduce a notion of time-consistency of a self-enforcing agreement, which is based on conditions of internal and external dynamic stability.

Further we focus on agreements, which satisfy condition of environmental efficiency. This property means that if any signatory withdraws from the coalition S and becomes a free-rider, total abatement undertaken by all players can only reduce. We assume that stepwise emission reduction of $(\mathbf{q}^S, \mathbf{q}^F)$, which constitutes Stackelberg equilibrium in the game $\Gamma_0(S)$, is held according to the proposed time-consistent scheme. Having accomplished its obligations for the moment t, a signatory considers withdrawing from coalition S if its payoff as a signatory of S over the time period $[t,m)$ is smaller than its payoff as a free-rider from set $F \cup \{i\}$.

We show that properties of internal and external dynamic stability of the agreement are maintained under the proposed scheme of emission reduction and no player has incentives to change its membership status towards the agreement during $t = 1,\ldots,m-1$.

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Note on Logarithmic Transformation of Utilities: Proportional, Egalitarian and Utilitarian Bargaining and NTU Games Solutions

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Keywords: *Cooperative NTU games, Bargaining games, Proportional solution, Egalitarian solution, Utilitarian solution, Nash bargaining solution*

Abstract: *We discuss here what happens with some solutions of bargaining games and NTU games under logarithmic transformations of the utilities of players*

In Pechersky (2007) the proportional excess for NTU games was defined axiomatically, and then it was used to define the *status quo*-proportional solution to bargaining games and to extend it to the non-leveled NTU games possessing following properties:

(a) $V(S)$ is positively generated, i.e. $V(S) = (V(S) \bigcap \mathbf{R}_+^S) - \mathbf{R}_+^S$ and $V_+(S) = V(S) \bigcap \mathbf{R}_+^S$ is a compact set, and every ray $L_x = \{\lambda x : \lambda \geq 0\}$, $x \neq \mathbf{0} = (0,...,0)$ does not intersect the boundary of $V(S)$ more than once;

(b) $\mathbf{0}$ is an interior point of the set $V^\wedge(S) = V(S) \times \mathbf{R}^{N \setminus S}$.

The corresponding space of NTU games is denoted by CG_+.

Recall that the resulting proportional excess is defined by $h_S(V,x) = 1/\gamma(V(S),x^S)$, where $\gamma(W,y) = \inf(\lambda > 0 : y \in \lambda W)$ is the gauge (or the Minkowski gauge function) of a set W. Let us denote the nucleolus (corresponding to the proportional excess) of a game V by $N(V)$. Then if $(N,V) \in CG_+$ corresponds to a bargaining game (q,Q) in the sense that for some $q \in \mathbf{R}_{++}^N$, $V(S) = \{x \in \mathbf{R}^S : x_i \leq q_i$ for every $i \in S\}$, for any $S \neq N$, $q \in \operatorname{int} V(N)$, and $V(N) = \{x \in \mathbf{R}^N :$ there is $y \in Q$ such that

$x \le y\}$, and $V(N)$ is non-leveled, then $N(V) = \lambda q$, where λ is such that $\lambda q \in \partial V(N)$. This bargaining solution is called *status quo*-proportional.

This solution was extended to non-leveled NTU games, the corresponding solution being called configurationally proportional solution (cf. Pechersky, 2007).

We discuss here an interesting, although sufficiently simple question, what happens with these (and some others) solutions under logarithmic transformations of the utilities of players. It turns that under logarithmic transformation the proportional excess transforms into the egalitarian excess due to Kalai. Formally, let $N = \{1,...,n\}$, and $x \in \mathbf{R}^N_{++}$. Denote $\mathrm{LN}(x) = (\ln(x_1),...,\ln(x_n))$. For every game $(N,V) \in CG_+$ let $V_{++}(S) = V(S) \cap \mathbf{R}^N_{++}$. Define $\mathrm{LN}(V)$ to be a NTU game such that for every S

$$\mathrm{LN}(V)(S) = \mathrm{LN}(V_{++}(S)) \subset \mathbf{R}^S$$

(since $V_{++}(S)$ is normal ($\mathbf{0}$-comprehensive) $\mathrm{LN}(V_{++}(S))$ is comprehensive).

Recall that the egalitarian excess is defined (cf. Kalai, 1972, 1975) by:

$$e(x, V(S)) = \max\{\lambda \in \mathbf{R} : x + \lambda e \in V(S)\}$$

If $h(.,.)$ is the proportional excess, and $e(.,.)$ – egalitarian excess, then it is not difficult to prove that

$$e(\mathrm{LN}(x), \mathrm{LN}(V_{++}(S))) = \ln(h(x, V_{++}(S))).$$

Since the logarithmic transformation preserves the preference relations the following proposition, where $N(V)$ and $PN(V)$ denote the nucleolus and the prenucleolus of a game V, is obvious.

Proposition 1.

1) If R denotes the *status quo*-proportional solution, and E – egalitarian solution for bargaining games, then for every bargaining game (q,Q)

$$\mathrm{LN}(R(q,Q)) = E(\mathrm{LN}(q,Q)),$$

where $\mathrm{LN}(q,Q) = (\mathrm{LN}(q), \mathrm{LN}(Q))$.

2) $\mathrm{LN}(N_p(V)) = N_e(\mathrm{LN}(V))$.

3) $\mathrm{LN}(PN_p(V)) = PN_e(\mathrm{LN}(V))$,

where index p corresponds to proportional excess, and index e – to egalitarian excess.

This proposition allows to modify the geometric characterization of the proportional nucleolus and prenucleolus to the corresponding characterization of the corresponding egalitarian solutions. This characterization is analogous to the well-known geometric characterization of the nucleolus and prenucleolus due to Maschler-Peleg-Shapley (1979).

Clearly, the logarithmic transformations of utilities transforms the Nash bargaining solutions into the utilitarian solution. To be more formal, $\mathrm{LN}(NS(Q)) = US(\mathrm{LN}(Q))$, where NS stands for the Nash solution, and US – for the utilitarian solution

Under logarithmic transformation of utilities the configurationally proportional solution becomes a solution, which is closely related to Kalai-Samet (1985) egalitarian solution for NTU games. They coincide for two-person games. They differ for n-person NTU games ($n > 2$) in the definition of the dividends.

Roughly speaking, the dividends for Kalai-Samet solution are defined for player i as a sum of dividends, which player receives in any sub-coalition he participates in. For logarithmic transformation of the configurationally proportional solution the dividends are defined by the maximum of the sums of dividends, where maximum is taken over all increasing sequences of coalitions containing i.

The corresponding system of axioms for the latter solution is given.

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Linear Symmetric Values for Games with Externalities

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Keywords: *Coalition structure, Externalities, Partition function game*

This paper presents a decomposition of the space of games with externalities under the action of the symmetric group (i.e., the group of permutations of the set of players). We also indentify all irreducible subspaces that are relevant to the study of linear symmetric solutions – namely those that are isomorphic to the irreducible summands of the space of payoffs R^n.We then use such decomposition to obtain our main result: a formula for all symmetric efficient values for games with externalities.

We study linear symmetric solutions using basic representation theory, the decomposition introduced in this paper is interesting and learns us more about games with externalities. Briefly, what we do is to derive direct sum decomposition of the space of games and the space of payoffs into elementary pieces. Moreover, any linear symmetric solution when restricted to any such elementary subspace is either zero or multimplication by a single scalar, regardless of the dimension of the elementary subspace; therefore, all linear symmetric solutions may be written as a sum of trivial maps.

Once we have such a global description of all linear and symmetric solutions, it is easy to understand the restrictions imposed by other conditions or axioms, for example, the efficiency axiom.

The reader will find here a different perspective than the more 'traditional' approaches to games with externalities. Besides presenting some results, one of the main objectives of the present work is to advertise representation theory as a natural tool for research in cooperative game theory.

Stable Cooperation

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Cooperation is a basic form of human behavior. And for many practical reasons it is important that cooperation remains stable on a time interval under consideration. There are three important aspects which must be taken into account when the problem of stability of long-range cooperative agreements is investigated.

1. *Time – consistency (dynamic stability) of the cooperative agreements*. Time-consistency involves the property that, as the cooperation develops cooperating partners are guided by the same optimality principle at each instant of time and hence do not possess incentives to deviate from the previously adopted cooperative behavior.

2. *Strategic stability*. The agreement is to be developed in such a manner that at least individual deviations from the cooperation by each partner will not give any advantage to the deviator. This means that the outcome of cooperative agreement must be attained in some Nash equilibrium, which will guarantee the strategic support of the cooperation.

3. *Irrational behavior proofness*. This aspect must be also taken in account since not always one can be sure that the partners will behave rational on a long time interval for which the cooperative agreement is valid. The partners involved in the cooperation must be sure that even in the worst case scenario they will not loose compared with non cooperative behavior.

The mathematical tool based on payoff distribution procedures (PDP) or imputation distribution procedures (IDP) is developed to deal with the above mentioned aspects of cooperation.

Many-agent Interaction in Multiperiodic Auctions with Variable Parameters

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Keywords: *Many-agent interaction, Nash equilibrium, Compromise profile, Dynamical sealed-bid auctions, First-price auctions*

Abstract: *Mathematical competitive multiperiodic repeated auctions models with many-agent interaction are considered. Dynamical repeated sealed-bid auctions with finite number of sellers and buyers are formalized and studied.*

Assume that number of periods during which auction repeats is T. Let's consider an initial model that is functioning over the first period. Let $N = \{1,...,n\}$ be denoted a finite set of sellers. The sellers simultaneously and independently of one another and of buyers display their lots for sale. A seller $i \in N$, having his lot valuation $r_i > 0$, declares lot price $r_i > 0$, $i = 1,...,n$, $y_i \in Y_i$ (here $Y_i = \{0,1,...,l_i\}$ – set of strategies of the seller of a lot *i*, l_i – a positive integer number). Let us denote by $M = \{1,...,m\}$ a finite set of buyers. The buyers simultaneously and independently of one another and of sellers declare their prices $x_{ji} > 0$ for every lot, having lot valuations $v_{ji} > 0$, $j = 1,...,m$, $i = 1,...,n$, $x_{ji} \in X_j$ (here $X_j = \{0,1,...,k_j\}$ – set of strategies of the buyer *j*, k_j – a positive integer number).

It is assumed that the number of sellers does not exceed the number of buyers ($n \le m$).A buyer *j* payoff (profit) h_{ji} when he gets a lot *i* is equal to the difference between v_{ji} and x_{ji}. A seller *i* payoff (profit) g_i is the difference between y_i and r_i. We also assume that every agent knows payoff functions of the other agents.

Suppose that ability of paying (solvency) of the buyer *j* is bounded by his budget constant $W_j > 0$, $W_j \le \sum_{i=1}^{n} v_{ji}$.

Assume that if the buyer *j* has won more than one lot he gets only *i-th* one which yields h_{ji} – the highest among the others: $h_{ji} = \max_{s \in S_j}(h_{js})$. Here S_j is the set of lots which the buyer *j* has won at a given stage: $x_{js} > \max_{\substack{1 \le k \le m \\ k \ne j}}(x_{ks})$. Buyers declaring the second highest price get the other lots.

If there are more than one buyer declaring the same price ($x_{ji} = x_{ki}$, $j \ne k$, $i = 1,...,n$ and $j,k = 1,...,m$), they play the auction's round between them at the next step.

The process is repeated at the next step with $n - p$ sellers and $m - c$ buyers. Here *p* is the number of sold lots, *c* is the number of buyers with exhausted budget.

The first stage of the process is completed if all lots are sold. At the next stage the parameters of the auction model Γ^0 change depending on strategies used by the agents at the first stage. So we get a new auction model $F(\Gamma_s^0)$, where $s = s(x_{ji}, y_i)$ is an agents strategy profile used by agents at the first stage of the auction process, F is the transition operator.

Each agent tries to maximize his profit got in the course of the *T* stages of the many step auction process. Cournot-Nash equilibrium points and compromise solutions are found by means of numerical methods.

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Optimal Stopping of a Risk Process with Disruption and Interest Rates

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Keywords: *Risk process, Disorder problem, Dynamic programming, Interest rates, Optimal stopping*

Abstract: *A problem of determining the optimal stopping time of the risk process as well as the corresponding payoff in the model with interest rates and disruption is examined. The solution is derived in the model which allows for change in distributions of random variables representing claim amounts and inter-occurrence times between losses according to some unobservable process. Possible applications of the investigated problem to insurance practice as well as references to previously examined models are presented.*

1 Introduction

The following model has been often investigated in collective risk theory. An insurance company with initial capital u receives premiums at constant rate $c > 0$ and pays for claims which occur according to a point process at times $0 < T_1 < T_2 < \ldots, \lim T_n = \infty$. The risk process $(U_t)_{t \in \mathbb{R}_+}$ is defined as the difference between the income and total amount of claims up to time t.

Many past articles have been concentrating on solving the problem of optimal stopping of the risk process in such models. In his classical work [3], Jensen provided a method of finding an optimal stopping time maximizing the expected net gain $\mathrm{E}(U_{\tau^*}) = \sup\{\mathrm{E}U_\tau : \tau \in \mathrm{C}\}$, ($\mathrm{C}$ was a class of feasible stopping times) using smooth semi-martingale representation of the risk process.

Such approach turned out to be unprofitable when some utility function of the risk process had to be considered. In [1] the authors solved the optimal stopping problem applying dynamic programming methodology. Muciek made an effort to adapt this solution to insurance practice introducing in [4] a model allowing for investment of the company's capital and increase of claim sizes at given interest rates.

Both mentioned models proved to be still quite restrictive as they enforced fixed distributions for random variables (r.v's) describing inter-occurrence times between losses and amounts of subsequent losses throughout the entire period of observation. As allowing for a disruption would make the derived stopping rule more interesting in terms of insurance practice, Ferenstein and Pasternak-Winiarski introduced a model in [2] in which the mentioned distributions changed according to some unobservable r.v. The main motivation for this research is to combine the findings of the models from [4] and [2] and determine the optimal stopping rule in the model allowing for the widest class of investigated processes.

2 The model and the optimal stopping problem

Let $(\Omega, \mathcal{F}, P)$ be a probability space on which we introduce following r.v.'s and processes:

1) Unobservable r.v. κ with values in $\mathbb{N}_0 = \mathbb{N} \cup \{0\}$, having geometrical distribution with parameters $p, \pi_0 \in [0,1]$:

$$P(\kappa = 0) = \pi_0,$$

$$P(\kappa = n) = (1-\pi_0)p(1-p)^{n-1}, n \in \mathbb{N}.$$

2) Claim counting process with jumps at times $0 < T_1 < T_2 < \ldots$.
3) A sequence of r.v.'s $S_n = T_n - T_{n-1}, n \in \mathbb{N}, T_0 = 0$, representing the inter-occurrence time between $n-1$ th and n th loss. S_n depends on the unobservable random time κ and is defined as follows:

$$S_n = \begin{cases} W_n', & if\ n \le k \\ W_n'', & if\ n > k \end{cases}$$

$W_n', n \in \mathbb{N}$ is a sequence of i.i.d. r.v.'s with cumulative distribution function (c.d.f.) F_1 (satisfying the condition $F_1(0) = 0$) and density function f_1. Similarly $W_n'', n \in \mathbb{N}$, forms a sequence of i.i.d. r.v.'s with c.d.f. F_2 ($F_2(0) = 0$) and density function f_2. We assume that f_1 and f_2 are commonly bounded by a constant $C \in \mathbb{R}_+$. Furthermore we impose that W_i' and W_j'' are independent for all $i, j \in \mathbb{N}_0$.

4) A sequence $X_n, n \in \mathbb{N}_0$ of r.v.'s representing successive losses. They also

depend on the r.v. κ:

$$X_n = \begin{cases} X'_n, & if\ n < k \\ X''_n, & if\ n \geq k \end{cases}$$

where $X'_n, n \in \mathbb{N}_0$ is a sequence of i.i.d. r.v.'s with c.d.f. H_1 ($H_1(0) = 0$) and density function h_1 whereas $X''_n, n \in \mathbb{N}_0$ forms a sequence of i.i.d. r.v.'s with c.d.f. H_2 ($H_2(0) = 0$) and density function h_2. X'_i and X''_j are independent for all $i, j \in \mathbb{N}_0$.

We assume that r.v.'s $W'_n, W''_n, X'_n, X''_n, \kappa$ are independent.

We introduce an interest rate at which we invest accrued capital (constant $\alpha \in [0,1]$) as well as rate $\beta \in [0,1]$ describing the growth of claims and define a capital assets model for the insurance company as

$$U_t = ue^{\alpha t} + \int_0^t ce^{\alpha(t-s)} ds + \sum_{i=0}^{N(t)} X_i e^{\beta T_i},\ X_0 = 0.$$

The return at time t is defined by the process

$$Z(t) = \begin{cases} g_1(U_t)\mathrm{I}_{\{U_s > 0, s < t\}} & if\ t < t_0 \\ 0 & if\ t \geq t_0 \end{cases}$$

where g_1 is a utility function and the constant t_0 is a fixed time which denotes the end of the investment period.

First we solve a finite horizon optimal stopping problem assuming a fixed maximal number of claims – K. We define a family of σ – fields generated by all events up to time $t > 0$:

$$\mathcal{F}(t) = \sigma(U_s, s \leq t) = \sigma(X_1, T_1, \ldots, X_{N(t)}, T_{N(t)}).$$

Let $\mathcal{T}$ be the set of all stopping times with respect to the family $\{\mathcal{F}(t)\}_{t>0}$. For $n = 0,1,2,\ldots k \leq K$ we denote by $\mathcal{T}_{n,K}$ such subsets of $\mathcal{T}$ that satisfy the condition

$$\tau \in \mathcal{T}_{n,K} \Leftrightarrow T_n \leq \tau \leq T_K \text{ a.s.}$$

We seek an optimal stopping time τ_K^*, such that

$$\mathrm{E}(Z(\tau_K^*)) = \sup\{\mathrm{E}(Z(\tau)) : \tau \in \mathcal{T}_{0,K}\}.$$

In order to find this optimal stopping time we first consider optimal stopping times $\tau^*_{n,K}$ such that

$$\mathrm{E}(Z(\tau^*_{n,K}) \mid \mathrm{F}_n) = ess\ sup\{\mathrm{E}(Z(\tau) \mid \mathrm{F}_n) : \tau \in \mathrm{F}_{n,K}\}$$

and then using the standard methods of dynamic programming we obtain $\tau^*_K = \tau^*_{0,K}$.

Finally we propose a method of solving the optimal stopping problem in this model in case of infinite horizon.

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Analysing Plural Normative Interpretations in Social Interactions

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Keywords: *Interpretation, Norm, Organisation, Real and formal power*

In this study, we deal with «problems with the law» which are not reducible to straightforward offences, and which have to do with the legitimate plurality of interpretations of a given normative system. Distinct individuals or organisations may favour divergent interpretations of the same norms or principles. But such a plurality does not preclude the possibility that norms or principles be considered effective. Indeed, their susceptibility to various interpretations might be viewed as a constitutive part of their ability to impose structure onto social interaction.

This constitutive role of the plurality of interpretations has been variously diagnosed in the social sciences (for ex. in the work of Reynaud [9] and Matland [5]) as well as in political and legal theory. But a unified theoretical framework and a systematic descriptive method are still lacking, although recent concepts and methods from political theory and game theory offer guidelines in this respect.

The article put forward methodological tools and a restricted set of concepts which enable us to offer a formal treatment of these matters. We aim at proposing useful analyses of the simultaneous and non-transitory co-existence of distinct interpretations of given norms and principles, in cases where it cannot simply be hypothesized that these norms or principles have lost validity.

We suggest that interpretative plurality can be thought of in terms of alternative systems of individual and coalitional power. Power, in turn, is analysed through the game-theoretic notion of an effectivity function. Using these tools, we next proceed to specify a general contrast between formal authority and real power in organisations, with the purpose of understanding how interpretative controversies might account for some documented phenomena of real-authority migration among social or political actors, with no change in the formal rules.

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One-dimensional Bargaining with a General Voting Rule

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Keywords: *Bargaining, Subgame perfect equilibrium, Voting rules*

Abstract: *We study a model of multilateral bargaining over social outcomes represented by points in the unit interval. An acceptance or rejection of a proposal is determined by a voting rule as represented by a collection of decisive coalitions.*
The focus of the paper is on the asymptotic behavior of subgame perfect equilibria in stationary strategies as the discount factor goes to one. We show that, along any sequence of stationary subgame perfect equilibria, as the discount factor goes to one, the social acceptance set collapses to a point. This point, called the bargaining outcome, is independent of the sequence of equilibria and is uniquely determined by the set of players, the utility functions, the recognition probabilities, and the voting rule. The central result of the paper is a characterization of the bargaining outcome as a unique zero of the characteristic equation.

This paper analyzes a model of multilateral bargaining where players must choose one alternative from a set of alternatives represented by points in the unit interval. An alternative might be a level of taxation, a location of a facility, or an index of an ideological content of a policy (left vs. right).

Bargaining proceeds as follows. At the beginning of each period, nature randomly selects one of the players as a proposer. The probability for a player to become a proposer, the so-called recognition probability, is assumed to be the same in each period. The player chosen by nature puts forward a proposal that specifies one alternative. All players then react to the proposal. Each player can either reject or accept the proposal. The votes are cast sequentially, the sequence of responses being fixed throughout the game. Whether the proposal passes or fails is then determined by a voting rule, as represented by a collection of decisive coalitions.

The passing of a proposal requires an approval of it by all the players in some decisive coalition. Examples of voting rules include the unanimity acceptance rule when a passing of a proposal requires an approval of it by all the players, the quota rule, when a fixed number of votes is needed for a passing of a proposal, or the simple majority rule. If the proposal passes, it is implemented and the game ends. In this case each player receives a discounted utility of the alternative. Otherwise, a new period begins.

We consider subgame perfect equilibria in stationary strategies. Stationarity means that a proposal of any player does not depend on the history of play and a player's reaction to a proposal depends only on the proposal itself. The focus of the paper is on the asymptotic behavior of stationary subgame perfect equilibria as the discount factor approaches one.

The results are as follows. We prove that, along any sequence of subgame perfect equilibria in stationary strategies the social acceptance set collapses to a point.

This point, called the bargaining outcome, is independent of the sequence of equilibria and is uniquely determined by the set of players, the utility functions, the recognition probabilities, and the voting rule. The central result of the paper is a characterization of the bargaining outcome as a unique zero of the characteristic equation. This paper is the first to provide the characterization of the limit of stationary equilibrium in a one-dimensional model of bargaining.

The results are obtained under rather minimal assumptions. Thus the instantaneous utility functions are only assumed to be single-peaked and concave. Furthermore, we require that the intersection of any two decisive coalitions contain a player with a positive recognition probability. This requirement puts but a very mild restriction on the voting rule and the recognition probabilities.

Mean Value of the Infinite Coalition Games

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Keywords: *Infinite coalition games, Mean win, Risk, Conflict*

Abstract: *The infinite coalition games are considered and studied from the point of view of risks and conflicts. These games are corresponded to Markov's chains. Study of the infinite coalition games is brought to study of Markov's chains. The mean win of these games is calculated.*

The infinite coalition games are considered in the report. The risk and conflicts of these games are introduced. The algorithm is suggested constructing a game without conflicts. It is shown how to correspond any coalition games to a Markov's chain. The study of these games can be reduced to the study of the Markov's chain. A mean gain and algorithm of its calculation is suggested.

Let us introduce a characteristic function $v(x_S)$ where $x_S = (x_1, \cdot, x_N)$ with the coordinates $x_i = 0$ or $x_i = 1$ depending on the fact if the *i*-th player belongs to the S-th coalition or not. We can define the distance between coalitions S_1 and S_2.

Definition We will call a coalition S the riskiest coalition with a radius of risk region h_{max} if the following optimization problem

$$\psi(S,h) = \frac{v(x_S + h) - v(x_S)}{\| h \|} \to min_{S,h}$$

reaches its minimum on S.

One of the ways for reducing of the risk is changing the function $v(.)$ that we discuss later about.

Conflicts happen when $v(S) + v(T) > v(S \cup T)$ for some coalitions S and T. How can we find the conflict coalitions?

It is suggested constructing *a lower superadditive hull* of the function $v(.)$ which we denote by $\tilde{v}(.)$. Let us define $\tilde{v}(.)$ as

$$\tilde{v}(S) = max_{S_1,S_2,\ldots,S_k}(v(S_1)+v(S_2)+\ldots+v(S_k),v(S)) \ , \qquad (3)$$

where maximum is taken for all subcoalitions $S_i \subset S, S_i \cap S_j = \varnothing, i \neq j$, $i,j \in \mathrm{N}$, N is the set of the natural numbers. The union of S_i is not all S necessarily. Obviously, if $v(.)$ is not negative and defined for all S_i then maximum is reached for S_i which union is equal to S.

Theorem 1. *The function* $\tilde{v}(.)$ *is superadditive.*

It is not difficult to find the conflict coalitions using the function $\tilde{v}(.)$. Moreover, we can find not conflict coalitions that were not defined before.

We consider the more general case when the probability of taking a part in a coalition M of *i*-th player is equal to $p_i(M)$ and not taking a part is equal to $q_i(M)$. Clearly, $p_i(M)+q_i(M)=1$. By definition we consider that the players make their decisions independently. Let us define for any coalition M a characteristic function $v(M)$. Calculate the probability of conversion a coalition M_2 into a coalition M_1. We will denote a number of players in the coalitions M_1 and M_2 by $|M_1|$ and $|M_2|$ correspondently. For conversion the coalition M_2 into coalition M_1 it is necessary to add N_1 players to M_1 and to remove N_2 players from M_2. Moreover, we have to retain $N_2 = |M_2| - N_2$ players in the coalition M_2. It is not difficult to calculate the probability of these operations.

Let a number of all coalitions be equal to K. It is possible that the coalitions have equal parity. It means that the probability to choose one of them equals $frac1K$. If the coalitions have unequal parities and we are in the coalition M_2, then we denote the probability of choosing M_1 by $p(M_2,M_1)$. Then the probability of transition from M_2 to M_1 is equal to multiplication of all probabilities.

If we consider all coalitions and their probabilities of transitions to other coalitions, then we get the Markov's chain. Correspondence between the coalition game and the Markov's chain is one to one. To find the mean value of the gain we have to study its Markov's chain. Every Markov's game can be represented by a graph Γ_V. We will call Γ_V *the graph of the coalition game* V.

To find the mean gain of the game we have to represent the Markov's chain as an union of one or several ergodic classes and also periodic classes (some classes may be empty). The formulas for calculation of the mean gain of the game are given for every possible case.

Journals in Game Theory

GAME AND ECONOMIC BEHAVIORTHEORY REVIEW

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Rent-seeking Contests of Weakly Heterogeneous Players with Interdependent Preferences

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Keywords: *Contest theory, Heterogeneous interdependent preferences, Rrent-seeking*

There is now a large and growing literature on contest theory. A contest is a game where players simultaneously exert effort to increase the chance of winning a prize. One special field of the contest literature is rent-seeking. Rent seeking generally implies the extraction of uncompensated value without making any contribution to productivity. For example it is held to be associated with firms efforts for lobbying to cause a redistribution of the government spending allocation.

In this paper we explore rent seeking contests (see Nitzan (1994) and Tollison (1997)), where the set of players is weak heterogeneous concerning their preferences. The players have negatively interdependent preferences in different parameter values. For a long time the assumption of preferences being independent from other players' payoff has been unquestioned in economic. In the more recent literature especially the literature about evolutionary game theory it became more common to relax this assumption of purely self-interested preferences. Hehenkamp et al. (2004) show that in rent-seeking contests, as analyzed here, it is evolutionary stable to behave like a player with interdependent preferences. Nevertheless the idea of interdependent preferences is rather old. Duesenberry (1949) laid a first empirical basis of interdependent preferences. More recent empirical support of the interdependent preference hypothesis is given by Bush (1994), Kapteyn et al. (1997), Levine (1998) and Davis and Holt (1993).

In the present paper we analyze contests, in which interdependent preferences of players are close but still distinguishable, using a simple but powerful technique of linear approximation. Its basic idea is that efforts of weakly heterogeneous players in equilibrium are close to those of homogeneous players. The equilibrium of identical players is well characterized, and the behavior of weakly heterogeneous players can then be analyzed by Taylor-expanding the corresponding function around the homogeneous equilibrium.

In general it is interesting to know how robust the symmetric equilibrium is with respect to weak heterogeneity. In contests, the elasticity of effort with respect to a player's own preference is large, which implies that weakly heterogeneous players respond strongly to their relative advantage or disadvantage.

In the paper we stress the importance of the exact preferences in order to understand the impact of the technology of rent-seeking on the structure of the outcome of the game. The arising question is whether a small amount of heterogeneity of players due to their different preferences is capable of causing changes in the equilibrium structure. Indeed, heterogeneity due to different preferences alter the structure of the equilibrium with respect to the question who drops out first. A second question is whether the chosen technology has an impact on the structure of the outcome. We calculate the threshold values of the discriminatory power for which players sequentially drop out. These threshold values are higher compared to those identified for player with independent preferences. Therefore, the technology parameter of rent-seeking contests has an impact on the structure of the outcome. The shown results are somehow intuitive, but in this paper the analytical way to derive those results is the main point.

On a Game of Women and Cats versus Men and Mice

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Keywords: *Dynamic game, Two person zero sum game*

Abstract: *This two-person game was described in "Stochastic models for Learning" by D. Luce and H. Raiffa (New York, Wiley, 1957). The game was introduced by D. Blackwell in 1954.*

We give an explicit formula for the value of this game and the optimal strategies of both players.

Journals in Game Theory

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On Equilibria with Non-Additive Beliefs

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Keywords: *Equilibrium, Extensive games, Non-additive beliefs, Rationality, Uncertainty*

We investigate equilibrium concepts for normal and extensive form games that are based on non-additive beliefs and Choquet expected utility theory. Such equilibria allow a distinction between uncertainty and risk, and are proposed for games in which there is uncertainty that cannot be modelled readily by probabilities, in particular due to lack of mutual knowledge of rationality. In such games a player's attitude towards uncertainty can be captured through the non-additivity of his beliefs. These beliefs are thus modelled as capacities, i.e. normalised set functions that are montone but not necessarily finitely additive, and according to Choquet expected utility theory players act as to maximise expected utility with respect to capacities.

For normal form games this analysis gives rise to a new robustness analysis for Nash equilibria that differs both from perfection and risk-dominance arguments. For example, whether a mixed strategy Nash equilibrium is robust will depend on the specific game in question. It is straightforward to show that (in this sense) "robust" Nash equilibria exist under standard assumptions. We also provide epistemic foundations for such equilibria in terms of players' knowledge and their beliefs about their opponents. This result rests on a decomposition property of capacities that is derived from an axiomatic approach to updating non-additive beliefs in specific circumstances. Perhaps paradoxically, in providing epistemic foundations it is necessary to consider the updating problem even for one-shot games.

For extensive games this approach leads to an equilibrium concept that applies equilibrium reasoning only on the equilibrium path, but (under assumptions of complete ignorance and uncertainty aversion) maximin reasoning off the equilibrium path. The equilibrium path itself is endogenously determined in this way. This allows a consistent interpretation of deviations from the equilibrium path as deviations from rationality, and leads to strategy profiles that are equilibria (and, in a sense, subgame perfect) but may differ qualitatively from sequential equilibria.

Finally, we consider applications of these concepts.

Compliance Pervasion and the Dynamic of Norms: the Game of Deterrence Approach

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Keywords: *Compliance pervasion, Evolutionary stability, Game of deterrence, Norms, Playability*

1 Group behaviour and the dynamics of norms

The accelerated change of regulatory and legal corpuses resulting from globalization, or technological and scientific progress, impacts the norms prevailing within groups of individuals. While occurring at a macro level, this change orientates the management and the development of the firm. The stack of such changes increases the complexity of the norms system, implying more often than not that these changes occur without a prior comprehensive analysis of their effects. The consequences are that the new sets of norms resulting from the changes may contain inconsistencies, hence be weakly relevant to the goals pursued through modifying the existing set of norms. It follows that the expected degree of compliance of the group with the new set of norms may significantly decrease, putting the whole system in question

The existing literature contains many papers or books dealing with the economic effects of norms systems stemming from regulations and laws [11].

Analysis of norms systems has also been conducted in areas of management as different as quality management, organization, or human resources management. In particular anthropologists and psychologists have developed many experimental and theoretical studies to analyze the impact of cultural factors on the behaviour of individuals inside the firm [2, 5, 6, 12, 13].

On its side, Game Theory has been concerned with the effect of norms at different levels, individual or collective. For instance game theoretic tools have been applied to legal and argumentation issues [1, 8, 10]. On the other hand Game Theory has proposed formal models of cultural issues [2], and analyzed the pervasion of behaviours within a given population through an evolutionary approach [9].

Game Theory thus constitutes an efficient tool for apprehending compliance of behaviours with respect to the norms of a given group.

2 The game of deterrence approach

The present paper proposes a game theoretic method for analyzing compliance with norms at two different levels.

The first one can be called the individual / static level in that sense that it considers the behaviour of a single individual facing a set of norms. More precisely, this face-to-face will be represented as a game with two players: the individual which behaviour is subject to the analysis, and an abstract player representing the group to which the individual belongs. With this group is associated a set of norms, generating a set of group behaviours, in other words a set of group strategies, which efficiency in front of the individual's behaviours needs to be assessed.

To perform this assessment, we shall resort to matrix Games of Deterrence, a particular category of qualitative games analyzing threshold effects, through breaking down the set of states of the world into acceptable and unacceptable states. These games will thus enable to determine which non compliant strategies of the individual may be deterred by group strategies.

More precisely, it has been shown that with each matrix game of deterrence one can associate a bipartite graph called graph of deterrence, such that seven types of graphs can be distinguished, each one associated with a particular set of game solutions properties [7]. Through adopting this graph representation, it will be possible to assess the effect of changes in the graph structure on the set of game solutions properties. Then coming back to the matrix representation one will be able to define the required changes in the Group strategies that will produce such change of structure. Interpreted at the level of norms, this approach will enable to define the specific measures that should be taken by the group in order to protect its norms. On the other hand, it will determine if some of the existing norms should be changed in order to restore efficiency and consistency of the norms set.

The second level considered is a collective / dynamic level. Evolution of behaviours within the group will be assessed through using the strategies properties of a

Replicator Dynamic based evolutionary game of deterrence [9]. In particular, pervasion of compliance with norms will be assessed through determining the specific structure of the matrix supporting the evolutionary game, and hence the specific type of graph of deterrence associated with that game.

The analysis will thus enable to link the individual /static and the collective / dynamic levels. It will in particular point out how the impact on an individual of a specific set of norms will orientate the evolution of the group.

Applications to management will concern rules and motivations on the one hand, possible sanctions on the other.

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Competition between Two Ports: from Game Theory Perspective

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Keywords: *Competition, Cooperative game, Cournot model, Logit model, Non Cooperative game*

In this paper two ports in Pakistan are taken as a case study. There are four container terminals in two ports. 1. KPT (Public sector) 2. KICT (Private sector) 3. PICT (Private sector).These three container terminals are in one Port that is Karachi Port. Fourth player (container terminal) known as QICT (Private sector) is in second Port that is Port Qasim.

Player 1 is playing Cooperative game with players (2, 3):Reasons: Public sector does not charge for container handling or charge very low as compare to other three terminals, only Geared vessels i.e. vessels which have their own equipments to transfer containers come to KPT. Moreover KPT is not interested to catch the traffic because they do not have modern equipment to handle containers and from geared vessels they get nothing except wharfage charges. But if more traffic will come to private terminals it will increase the profit of KPT too because KPT has agreement with both Private sectors to give fixed amount from what they charge from containers vessels. KPT has also allowed both private sectors for further investment to increase the terminal areas and equipments.

Players 2 and 3 are playing Non-Cooperative game and Player 4 (Port Qasim) is also playing Non-Cooperative game with all three players (Karachi Port): They compete in two dimensions: a) The price (cargo handling charges) they charge and b) The service level they provide. As one example PICT got big ship (Mother vessel) from QICT. But QICT after decreasing their charges form 110 dollars to 40 dollars got the ship back.

PICT and KICT compete with each other in Karachi Port, since they are substitute, though imperfect of each other. On the other hand, KPT inclusive of both KICT and PICT has to compete against Qasim Port for business.

Although total traffic volume and container handling at Karachi port is larger than at Qasim port, but if we compare the performance of individual terminal, since last 4 years QICT is performing better than the three terminals of Karachi port. One way to compete with port Qasim is that all the terminals at Karachi port form collusion and merges as a single entity. Set joint low container handling prices and use each other facilities (e.g. use vacant berths of PICT due to low demand at this terminal in case of congestion, PICT is located close to Karachi as compare to Port Qasim, KPT has stores to de stuff containers etc), try to catch big vessels from QICT and share profit.

Cooperative Game: There are two points in a cooperative game (i) what is the payoff for each coalition? and (ii) what payoff each player in the coalition should get. To solve two problems, two other vital concepts may be introduced: The characteristic function (which assigns value to every possible coalition) and the core of n-player game (to show what each player could get). Cooperative Payoff Regions for Prisoners' Dilemma:

In case the players have the ability to write binding agreements during the pre-play negotiations, they need not confine their attention to Nash equilibria of the game to be played. They can agree to implement any payoff pair in the cooperative payoff region of the game. In cooperative game players can do a whole better than those confined to choosing Nash equilibria.

After cooperating with each other they will play Non-Cooperative game with second port that is QICT. Now in this case there are two players and is then said to be a duopoly. A French economist Cournot presented the idea of Nash equilibrium for this case more than a century ago. But he presented this idea for two firms who are competing by setting quantities. But in this situation two ports will compete in terms of price. Another French economist Bertrand had developed a model known as Bertrand Model in 1883. This model applies to the firms that choose prices instead of quantities and make their decision at the same time.

Homogenous services: Bertrand model can be applied if both ports offer homogeneous services. If these two duopolists compete by simultaneously choosing a price, then what price will each charge, and how much profit will each earn? Because the services are homogeneous, customer will select the port which will offer lowest price. If both charge the same price, customer will be indifferent as to which port to select and each port will capture the half market. Nash equilibrium in this case is the competitive outcome i.e., both set price equal to marginal cost. Since price equals marginal cost, both earn zero profit.

Differentiated Services: But services offered by two ports are different (due to location, quality of service, availability of equipment etc). Now if both ports set their price at the same time, we can use the Cournot model to determine the resulting equilibrium.

Logit Model: In order to find the Nash equilibrium mathematically, logit model will be developed in which both ports may be defined by set of certain attributes which make them attractive for customers. Port Qasim is attractive for the customers due to low handling and wharfage charges and other facilities provided by the port authorities. Private terminals are attractive due to fast handling rate as compare to Public Sector.

Thus utility of each port is a linear function of two variables: Price and Handling rate. Market share of ports can be calculated with the help of logit utilities of these two ports.

A Dynamic Procurement Auction with Persistent Backlog and Capacity Constraints

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Keywords: *Dynamic games, Procurement, Asymmetric auctions, Backlog, Capacity constraints, Bid preferences*

This paper contributes to the literature on dynamic games of asymmetric information by formulating and numerically solving for the Markov Perfect Equilibrium (MPE) of an infinite horizon dynamic procurement auction. In the model, bidder asymmetry is generated and evolves endogenously because the bidding firms are subject to capacity constraints and their backlog of work may persist for several periods. A firm's costs are probabilistically increasing in its capacity utilization rate – the ratio of the firm's backlog to its capacity. At a MPE, firms inflate their bids in accordance with the option value of a contract because winning a project in the current period implies higher costs as well as more committed capacity in the future. Also, a firm's value function is found to be decreasing in its backlog and increasing in its capacity and its rival's backlog. Keeping the average contract size fixed, we investigate whether the procurer can reduce its procurement costs by holding infrequent auctions with lumpy project sizes or frequent auctions with smaller project sizes. It is found that the latter policy leads to lower costs of procurement. The intuition is that bidder asymmetry is costly for the procurer and lumpy contract allocation leads to sequences of auctions where bidders are more asymmetric on average than less lumpy policies. We adapt the setup to incorporate bid preferences. Preliminary results indicate that the optimal rate of bid preference is zero. We also obtain a cost ranking for the first price and second price auction formats.

A Feasible Proposition for Cost-allocation in Cooperative Markets

Jan Selders and **Karl-Martin Ehrhart**

Keywords: *Application: financial management, Application: strategic management, Cooperative games, Cooperative markets, Cost-allocation*

Cost-allocation problems within organizations or projects of voluntary cooperation usually incorporate the characteristics of coalition games which are appropriately analyzed by cooperative game theoretical methods. Within this theoretical framework, the application of famous solution concepts, like the core or the Shapley-Value, requires the use of the characteristic function, which assigns to each player and each coalition of players their stand-alone outcome. This requirement, however, imposes severe restrictions to the application to real questions in practice, because the problem becomes very complex if the number of players is large. Therefore, it is not astonishing that these concepts have been hardly ever applied in practice.

In our paper we present an alternative approach of a feasible cost-allocation method that fulfills the game-theoretic criterion of stability, which ensures that each player and every coalition has an incentive to participate in the game. For this purpose, we do not model the game as a game in coalitional form, but as a network. Setting it up as a matrix similar to an input-output analysis, a line of the matrix corresponds to the commodities a player delivers to his peers, while the corresponding column lists all the commodities received from the other players. To allocate costs for the produced and traded commodities, we propose to use a price system.

One major advantage of the network model is that the amount of players and the amount of commodities are not restricted and do not diminish applicability. We therefore examine a market with an unlimited number of players that can be consumers and producers at the same time. Since the market possesses all the characteristics of division of labor, we define the players to be exclusive holders over their technologies (even if they are so only because of an agreement with the other players). We allocate costs through a price system for any number of multi-output monopolies that trade an arbitrary number of commodities. To allow for a realistic model, we also consider that all technologies are interdependent, that is, the prices set by one player effect all cost-functions – and therefore all prices – of all other players.

Since we acknowledge that our (internal or external) market (in the broader sense) is a cooperative one, we focus on stability instead of incentive compatibility. To determine prices that induce stability in such a setting, we introduce subsidy-free prices. Prices of a multi-output monopoly are subsidy-free, if they exactly cover costs and are free of cross-subsidization[1]. These conditions are described and illustrated formally. They challenge certain characteristics of the cost-function, which are described in detail.[2] Most importantly, to allow for subsidy-free prices, the (positive and monotonous) cost-function has to allow for non-rising average costs. Hence, the multi-output monopoly has to have a cost-function that defines it as a natural monopoly (within the context of the cooperation).

If the production technology of a player does not allow for a natural monopoly, the other players have an incentive not to share this technology jointly and to exclude the player from the game, since each player would be better off by accounting for his own needs. If the production of the collective demand is more expensive than the partitioned production of subsets of the collective demand, any coalition can attack the player's position by rejecting his commodities and producing them by itself. Subsidy-free prices do not allow such attacks and make entry into the monopoly unattractive for any other player or coalition and, therefore, guarantee that all other players channel their demand onto this one technology. They ensure that the least overall costs for all players are achieved by sharing every party's demand from the same source. Thus, subsidy-free prices effectuate the grand coalition.[3]

We generalize the theory of subsidy-free prices to model a cooperation of multiple, interdependent multi-output monopolies. Given the demand of every player, we determine the demand of any coalition. We then compare the costs the coalition has to bear in the game with its stand-alone costs. The costs the coalition has to bear in the

[1] The concept of cross-subsidization in enterprises was pioneered by Faulhaber, *Cross-Subsidization*, 1975.

[2] They were first described by Sharkey/Telser, *Supportable Cost Functions for the Multiproduct Firm*, 1978, who price commodities of a single regulated multi-output monopoly.

[3] The described correlation can be shown formally. Moulin, *Axioms of Cooperative Decision Making*, 1988, proves that stability in a coalitional cost-sharing game is equal to the underlying cost-function being supportable, in case of one single source of supply (one multi-output monopoly).

game are determined by the prices it is charged by the other players. The stand-alone costs are the costs of a coalition if it does not participate in the game, meaning that it produces its demand by itself. If the charged prices are smaller than the stand-alone costs, it is worthwhile for the coalition to cooperate with the other players. If this holds for all coalitions, there exists no subsidization between the players – no player pays more to fulfill his demand than he should –, and everybody is better off by forming the grand coalition that produces the aggregate demand of all players.

This is always the case if the prices that are used to allocate costs are subsidy-free. In order to prove this proposition, in our paper we show that for our model the property of subsidy-free prices is equivalent to the core solution in the corresponding game in coalitional form and that the existence of a system of positive subsidy-free prices is guaranteed. Therefore, to check that indeed the proposed method of cost allocation is stable, one does not have to set up a characteristic function and examine each and every possible coalition, but one only needs to check if all prices are subsidy-free. It is this characteristic of our approach that guarantees its feasibility.

Summarizing, we introduce a feasible solution to cost-allocation problems in any given setting, where any number of players can act as consumers, as producers, or as both at the same time. Our proposed and existing solution offers the same benefits as the core in games in coalitional form. One can think of such diversified applications of this method as joint-ventures, management accounting, or pricing of public goods.

Differential Games with Random Duration in Resources and Environmental Economics

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Keywords: *Differential games, Resource economics, Environmental economics, Cooperation, Random duration, Shapley value, Imputation distribution procedure, Time-consistency*

Abstract: *We investigate a game-theoretical model of pollution cost reduction and a game-theoretical model of nonrenewable resource extraction under condition of random game duration. We find feedback Nash equilibrium with the help of Hamilton-Hacobi-Bellman equation derived by Shevkoplyas E. for the problem with random duration. Then we consider cooperative form of these games. Using "Nash equilibrium approach" for characteristic function construction we get Shapley Value in our concrete models. "Nash equilibrium approach" firstly was proposed by Petrosjan L.A. and Zaccour G. and it means that if k players form coalition K then the remaining players stick to their feedback Nash strategies. We study a problem of time-consistency for Shapley Value under condition of random game duration for both game-theoretical models. We apply theoretical results earlier obtained by Petrosjan L. A. and Shevkoplyas E. for cooperative differential games with random duration.*

We consider 2 examples of differential games in resources and environmental economics under condition of a random game duration: the first is a game-theoretical model of pollution cost reduction [3] and the second is a game-theoretical model of nonrenewable resource exploitation [1], [2]. We find Nash equilibrium with the help of Hamilton-Jacobi-Bellman equation for differential games with random duration derived by Shevkoplyas E. in [5].

Then we consider cooperative form of the games. We calculate Shapley Value in both examples using "Nash equilibrium approach" proposed by Petrosjan L.A. and Zaccour G. in [3]. According to this method we make an assumption that left-out of coalition players stick to their feedback Nash strategies (they wouldn't form an anti-coalition).

The main research of the paper is devoted to a time-consistency problem of the Shapley Value in our concrete games with random duration. Time-consistency condition means that at the each time instant each player wouldn't have a reason to deviate from a cooperative solution. The analytic form of time-consistency condition in cooperative differential games with random duration was derived by Petrosjan L.A. and Shevkoplyas E. in [4]. It was proved that in many cases Shapley Value is not time-consistent. We can solve this problem with the help of special payoffs at the each time instant (or imputation distribution procedure).

Now we apply theoretical results to our examples of differential games with random duration.

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The Simplified Modified Nucleolus and the Bankruptcy Problem

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Keywords: *Bankpuptcy problem, The SM-nucleolus*

In the paper we describe a bankruptcy problem following the treatment in the two-thousand-year-old Babylonian Talmud. The *bankruptcy problem* is a distribution problem involving the allocation of a given amount of a perfectly divisible good among a group of agents. The focus is on the case where the amount is insufficient to satisfy all their demands. The story is the following: «A person dies leaving a number of debts that total more than the size of the estate». The question is: how should the estate be divided among the creditors?

Problems of the bankruptcy type arise in many real life situations. The classical example would be that of a bankrupt firm that is to be liquidated. Another example would be the division of an estate amongst several heirs, particularly when the estate cannot meet all the deceased's commitments.

Generally speaking the bankruptcy problem is not a game in strict meaning of this word. To get round this difficulty we define a game called the bankruptcy game corresponding to the bankruptcy problem [1]. Having this game we search for its solution using the simplified modified nucleolus (the *SM*-nucleolus) as a solution concept [5].

There are at least three simple methods for solving bankruptcy problems in practice, but each is deficient in one or more ways. Here we propose the *SM*-nucleolus as a distribution rule in bankruptcy games.

For arbitrary but fixed values of debts $d_1,\ldots,d_n$ and estate E we find the *SM*-nucleolus in two ways: analytically and by means of the computer program realized in the Delphi environment. Moreover, in case the estate value changes we find that the *SM*-nucleolus depends on this change and describe it in an analytical form.

Comparing the *SM*-nucleolus with other solution concepts ([2], [3], [4]) with respect to the bankruptcy game we obtain some interesting results. It is easy to show how the *SM*-nucleolus is connected with the nucleolus in case the estate value E changes, but debts are fixed.

Moreover, we show that the interval of all possible values E splits up into several parts and the *SM*-nucleolus, the nucleolus and the Shapley value coincide in the boundary points of these subintervals.

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Geometric Generalization of the Bargaining Problem and Some other Issues

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Keywords: *Bargaining, Geometry and compact spaces*

We will discuss bargaining by three or more players as well as via auctions, the dynamic of bargaining where the element of bounded time is important. The mathematical discussion will include the application of Tychonoff compactness theorem of products of compact spaces. This paradigm will eliminate the need for randomization assumption which came under criticism recently. Some application which will ensue from this theory will be discussed.

Learning, Testing, and Probability

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Keywords: *Bayesian updating, Expert testing*

We study the problem of testing an expert under the assumption that the true data-generating process has a learnable and predictive parametric representation, as do standard processes used in Bayesian statistics such as exchangeable and Markov processes. We design a test in which the expert is initially required to submit a date T by which he will have learned enough to e ectively approximate the true parameter. At time T he is expected to deliver speci c predictions about frequencies along a subsequence. His forecasts are then tested according to a simple hypothesis test. We show that this test passes an expert who knows the true data-generating process and cannot be manipulated by an uninformed one.

Dynamical Network Games with Changing Parameters and Vector Payoff Functions[1]

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Keywords: *Nash equilibrium, Compromise solution, Dynamical multistage game model, Many–agent interaction, Network games*

Abstract: *A dynamical multistage game model with changing parameters G1 of many–agent interaction with vector payoff functions in competitive network is constructed and studied in the paper.*
The game model G1 functionates over T periods. At each period the game model proceeds as the game model G changing its parameters when passing from a stage to the next one. The total agent's payoff in the game model G1 is defined as the sum of his partial payoffs gained at all stages of the game model.
Algorithms are given to find an equilibrium profile and compromise solution in the game G1.
Numerical examples are solved.

1. A static game model G of many–agent interaction in competitive network $\{N\}$ is constructed. In the frame of this model each agent $x \in X$ chooses a set $\Gamma^{+}(x)$ of his counterparts with whom he wishes to interact. Besides that he chooses a set $\Gamma^{-}(x)$ of agents against interactions with whom he has no objections. Here $X = \{x_1, x_2, \ldots, x_n\} = \{x\}$ denotes a set of nodes of the network $\{N\}$, where each node corresponds to an agent of the game model G . So, an agent x strategy in the game model G is the pair $\Gamma^{+}(x) \cup \Gamma^{-}(x) = \varphi(x)$. An ordered set of agents strategies $\varphi_G = (\varphi_G(x_1), \varphi_G(x_2), \ldots, \varphi_G(x_n))$ is called a strategy profile in the game model G. Each strategy profile in the game model G is characterized by connections between agents, number of three-agent coalitions, formed in the process of interaction between agents and by agent's vector payoff functions. The agent vector payoff function depends on number of coalitions into which he is included. Here a coalition $S = (x_{i_1}, x_{i_2}, x_{i_3}) \subset X$ is formed if each its member wishes

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to cooperate with his counterparts from this set $S=(x_{i_1},x_{i_2},x_{i_3})\subset X$ and simultaneously has no objections against theirs wishes to cooperate. Let's enumerate three-agents coalitions formed in the profile φ_G as follows: $S_\varphi^1, S_\varphi^2 ... S_\varphi^{n_x}$. Denote the set of these coalitions by S_3. Agents of the coalition $S_\varphi^j \in S_3$ have vector payoff functions $H_{x^1}(S_\varphi^j),...,H_{x^3}(S_\varphi^j)$, where $H_{x^k}(S_\varphi^j)=(H_{x^k}^1(S_\varphi^j)...H_{x^k}^3(S_\varphi^j))$. The total payoff $H_{x_k}(\varphi_G)$ of the agent x_k in the profile φ_G is presented by the sum of the payoffs gained by him in all coalitions into which he is included. The total payoff of the agent x_k in the game model G can be defined by different methods. Let's describe one of these methods. Let the set of coalitions into which the agent is included for the profile φ_G is as follows $S_\varphi(x)=\{S_\varphi^1(x),S_\varphi^2(x),...,S_\varphi^{n_x}(x)\}$.Besides that let the total profit of the coalition $S_\varphi^{i_l}(x)$ be $D(S_\varphi^{i_l}(x))$. Let the importance index of the agent x_k in the coalition $S_\varphi^{i_l}(x)$ be $\alpha_x(S_\varphi^{i_l}(x))$, so we have $\sum\limits_{x^{'}\in S_\varphi^{i_l}(x)}(\alpha_{x^{'}}(S_\varphi^{i_l}(x)))=1$. His profit in the coalition $S_\varphi^{i_l}(x)$ is equal to $\alpha_x(S_\varphi^{i_l}(x))\cdot D(S_\varphi^{i_l}(x))$. The total profit of the agent x in all tree –agent coalitions of the set $S_\varphi(x)$ is $\sum\limits_{i_l=1}^{n_x}\alpha_x(S_\varphi^{i_l}(x))\cdot D(S_\varphi^{i_l}(x))$. So we get the normal form game $\Gamma_1=\{X=\{x_1,x_2,...,x_n\},\Phi_G=\{\varphi_G^s\}_{s=1}^{2^{n(n-1)}},\{S_\varphi^{k_l}\}_{k_l=1}^{n_x},\{H_{x_k}(\varphi_G^s)\}_{x_k=1}^{n}\}$. The agent x tries to maximize his vector payoff function in the lexicographic sense. The compromise solution and Nash equilibrium profiles are found in the paper.

2. A dynamical multistage game model with changing parameters G_1 of many–agent interaction with vector payoff functions in competitive network $\{N\}$ is constructed in the paper.

The game model G_1 functionates over T periods. At each period $t=1,...,T$ the game model proceeds as the game model G changing its parameters when passing from a stage to the next one. The total agent's payoff in the game model G_1 is defined as the sum of his partial payoffs gained at all stages of the game model.

Algorithms are given to find an equilibrium profile and compromise solution in the game G_1.

Numerical examples are solved.

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Differential Inequalities for Dynamic Games

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Keywords: *Dynamic games, Guaranteed strategies, Pursuit evasion games*

In this paper we present a methodology to formulate closed-loop strategies for the players using differential inequalities [1]. In terms of pursuit evasion games our results represent a generalization of the results reported in [3]. Due to the space limitations we will present a guaranteed capture scenario as a representative example.

To approximate the minimum function from above, we propose the following function:

$$\bar{\sigma}_\delta(a_1,\ldots,a_N) = \left(\frac{N}{\sum_{i=1}^{N} a_i^{-\delta}} \right)^{1/\delta}, \ \delta > 0 \tag{1}$$

which converges from above to the exact minimum as δ approaches infinity. Similarly, to approximate the maximum function from above we propose the following function:

$$\bar{\rho}_\delta(a_1,\ldots,a_N) = \left(\sum_{i=1}^{N} a_i^{\delta} \right)^{1/\delta}, \ \delta > 0 \tag{2}$$

which converges from above to the exact maximum as δ goes to infinity.

Assume that we have N_e evaders and N_p pursuers and that the *i*-th evader's position is denoted as e_i and similarly that the *i*-th pursuer's position is denoted as p_i. Let us define the *collective positions* as $e = [e_1^T, \ldots, e_{N_e}^T]^T$ and $p = [p_1^T, \ldots, p_{N_p}^T]^T$. Following a similar idea as the one proposed in [3], we introduce the following functions:

$$\begin{aligned} \phi_\delta^i(e_i, p) &= \bar{\sigma}_\delta(\| e_i - p_1 \|, \ldots, \| e_i - p_{N_p} \|) \\ \pi_\delta(e, p) &= \bar{\rho}_\delta(\phi_\delta^i(e_1, p), \ldots, \phi_\delta^i(e_{N_e}, p)) \end{aligned} \tag{3}$$

Then, let us define $v(e, p) = \pi_\delta(e, p)$ and assume the following differential inequality:

$$\frac{dv(e,p)}{dt} = \frac{\partial v(e,p)}{\partial e} f_e(e, u_e^o(e,p)) + \frac{\partial v(e,p)}{\partial p} f_p(p, u_p^o(e,p)) \le g(e, p, v(e,p)) \tag{4}$$

where $g(\cdot,\cdot,\cdot)$ is a continuous function and $u_e^o(e,p)$ and $u_p^o(e,p)$ are respectively collections of the evaders and pursuers' strategies that maximize or minimize the time derivative, that is the growth, of the function $v(e,p)$ (for more details see [3]). $f_j(j, u_j^o(e,p))$, $j \in \{e, p\}$, represent collective dynamics of the evaders (when $j = e$) and the pursuers (when $j = p$) for the previously defined collective strategies.

By defining the capture of an evader to be accomplished whenever its Euclidean distance to any of the pursuers becomes less than a prescribed positive number *R* (also known as the "soft capture") we state the following theorem:

Theorem 1. Assume that the initial conditions $e_0 = e(t_0)$ and $p_0 = p(t_0)$ at the initial time t_0 are such that the players are outside of the capture regions defined by a positive number *R*, and that the maximal solution (as defined in [1]) of the following differential equation:

$$\frac{dw}{dt} = g(e(t), p(t), w), \quad w_0 = v_0(e_0, p_0) \tag{5}$$

is denoted as $\bar{w}(t, t_0, e_0, p_0, w_0)$ along the trajectories of the players' dynamic systems for the collections of their strategies $u_e^o(e,p)$ and $u_p^o(e,p)$. Then the capture of all evaders is guaranteed when pursuers use collective strategies provided in a vector form as $u_p^o(e,p)$, within a finite time *T* for any feedback strategies of the evaders if $\bar{w}(T, t_0, e_0, p_0, w_0) < R$.

An instrumental result in proving Theorem 1 is provided as Theorem 3.1.5 in [2]. In our final paper we will present conditions for guaranteed evasion based on differential inequalities too. We will also show that the proposed methodology would provide closed-form solutions for a class of games where the players are represented by nonholonomic nonlinear dynamic systems. Finally, by defining goals of the players in terms of trajectories of the system either reaching or not reaching the target sets, we will show how the proposed methodology can be applied to a larger class of dynamic games.

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Value Function of Differential Game with Simple Dynamics and Piecewise Linear Data[1]

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Keywords: *Differential games, Value function, Hamilton-Jacobi equation, Minimax solution, Hopf formulas*

Abstract: *A differential game with simple motions is considered under assumptions that the Hamiltonian and the cost terminal function are piecewise linear and positively homogeneous. The structure of the value function of the differential game is investigated in the framework of the theory of minimax (or/and viscosity) solutions for Hamilton-Jacobi equations. Inequalities are provided to estimate the value function. Cases of explicit formulas for the value function are pointed out.*

The report deals with a differential game with simple dynamics:

$$\dot{x} = f(u,v)\,, \tag{1}$$

$$t \in [0,1], \quad x \in R^n, \quad u \in P \subset R^p, \quad v \in Q \subset R^q,$$

where P and Q are compact sets. The function $f : P \times Q \to R^n$ is continuous and satisfies the condition

$$\min_{u \in P} \max_{v \in Q} \langle s, f(u,v) \rangle = \max_{v \in Q} \min_{u \in P} \langle s, f(u,v) \rangle = H(s), \quad s \in R^n. \tag{2}$$

A terminal cost functional

$$x(\cdot) \to \sigma(x(1)) \tag{3}$$

is given, where the function $\sigma : R^n \to R$ is Lipschitz continuous and positively homogeneous.

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It is known [1] that the value $\omega(t_0,x_0)$ exists in the differential game (1)-(3) at any initial state $(t_0,x_0)\in[0,1]\times R^n$. The value function $\omega:[0,1]\times R^n\to R$ is the unique minimax (and/or viscosity) solution [2, 3] of the following Hamilton-Jacobi equation

$$\partial\omega(t,x)/\partial t + H(\partial\omega(t,x))/\partial x = 0, \quad t\in(0,1),\ x\in R^n, \tag{4}$$

satisfying boundary condition

$$\omega(1,x)=\sigma(x), \quad x\in R^n. \tag{5}$$

As $H(\cdot)$ or $\sigma(\cdot)$ is convex or concave, the value function can be represented explicitly with the help of Hopf-Lax or Pshenichnyi-Sagaidak formulae [4, 5].

Explicit formulas for the value function are not known in general case.

The structure of the value function is considered below under assumptions on piecewise linearity of the Hamiltonian $H(\cdot)$ and the cost function $\sigma(\cdot)$.

It is known [6] that he positively homogeneous piecewise linear Hamiltonian can be written in the form

$$H(s)=\max_{q\in K}\langle s,q\rangle+\min_{p\in L}\langle s,p\rangle, \quad s\in R^n \tag{6}$$

where $K=\underset{i\in I}{co}\{q_i\}$, $L=\underset{j\in J}{co}\{p_j\}$ are convex polytopes.

It is proved that the value function of the game (1)-(3) satisfies the inequalities

$$\max_{i\in I}\min_{p\in L}(1-t)\sigma\left(\frac{x}{1-t}+p+q_i\right)\le\omega(t,x)\le\min_{j\in J}\max_{q\in K}(1-t)\sigma\left(\frac{x}{1-t}+p_j+q\right). \tag{7}$$

The cases are pointed out when the inequalities (7) provide explicit formulas for the value function ω.

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On Core Stability, Vital Coalitions and Extendability

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Keywords: *TU games, Core stability, Extendability*

The core of a cooperative game is called stable if it is a stable set in the sense of von Neumann and Morgenstern (1953). In this paper we restrict out attention to TU games. Several sufficient conditions for core stability may be found in the literature. For details see, e.g., van Gellekom, Potters, and Reijnierse (1999). A weak and simple sufficient condition, introduced by Kikuta and Shapley (1986) is called extendability. A TU game is extendable if each core element of any subgame may be extended to a core element of the entire game. The main part of the present paper is devoted to relaxing extendability in such a way that the modified extendability properties (1) are still sufficient conditions and (2) become necessary conditions for core stability when restricting the attention to some nontrivial important classes of games. We show that the game has a stable core if certain coalitions are extendable, namely those that are vital in the sense of Gillies (1959) and exact in the sense of Shapley (1971). For some classes of games, e.g., for the class of symmetric games (see Biswas, Parthasarathy, Potters, and Voorneveld (1999)), necessary and sufficient conditions for core stability have been found. We show that vital-exact extendability is also a necessary condition for core stability for three important classes of games: Assignment games, minimum coloring games, and simple flow games. Moreover, our approach enables us to reprove two characterization results of Solymosi and Raghavan (2001) and Bietenhader and Okamoto (2006) in a simple way.

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Optimal Contract Formation in International Agency Lease Arrangement

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Keywords: *Agency contract, Agency lease, Fee, Optimal contract, Profit*

Abstract: *In the paper, «international agency lease» scheme is considered. This scheme implies that there exists a leasing company (hereafter referred to as leaser) in the country «A», which intends to put some object on lease to some company from the country «B» (hereafter referred to as lessee), which needs this object for professional use. The leaser hires an agency company (hereafter referred to as agent) from the country «B» in order to find the lessee in the country «B». Leaser concludes the agency contract with the agent. The agency contract legally stipulates the fee rate offered to the agent by the leaser for the effort exerted by the agent in order to find the lessee. In the model the contract is considered to be formed if the values of two parameters are found: the fee paid to the agent regardless of whether he finds the lessee or not and the profit share paid to the agent in case the lease agreement has been concluded between the leaser and lessee. Therefore, the problem of optimal contract construction arises. In this model, the contract is considered optimal if the level of effort exerted by the agent provides the leaser's profit maximization in the deal. As a result, the leaser's profit has been calculated and the problem of optimal contract construction solved.*

Aircrafts and aircraft engines consist the group of most widespread objects of international lease arrangements. According to information provided by Boeing, the world aircraft fleet is up to 30 percent completed of leased aircrafts.

Leasing deal is considered national in case the leaser and the lessee (airline, in case of aircraft leasing) belong to one country. In case the leaser does not belong to the country of residency of the lessee – the airline operating the leased fleet -, the leasing deal is to be defined as international one.

One of the possible schemes of international lease arrangement is «international agency lease». This scheme implies that there exists a leasing company (hereafter referred to as *leaser*) in the country «A», which intends to put some object on lease to some company from the country «B» (hereafter referred to as *lessee*), which needs this object for professional use. The *leaser* hires an agency company (hereafter referred to as *agent*) from the country «B» in order to find the *lessee* in the country «B».

We can clearly distinguish the «principal – agent»-type relationships in the aforedescribed scheme of leasing deal arrangement. The goal of this research is the formation of optimal contract between the leaser (the principal) and the agent. The agency contract legally stipulates the fee rate offered to the *agent* by the *leaser* for the effort the *agent* exerts in order to find the *lessee*. In case this level of effort provides the *leaser's* profit maximization in the deal, the contract is considered optimal.

The model studied implies that the leaser chooses one of the three possible alternatives of agency leasing deal structuring: operating leases with the purchase option, operating leases without the purchase option, financial leases with the purchase option of leasing assets. The purchase option is understood as the right of lessee to buy out the leasing assets at the expiration of the leasing agreement. It is considered in the frameworks of this model that the lessee always exercises this right. The solution of the problem of optimal contract formation for each of the alternatives described allows us to find out which of the variants of international agency lease structuring is the most profitable for the leaser.

In the model, the contract is considered to be formed if the values of two parameters are found: r and α, where r - the fee paid to the *agent,* the fixed part, and α - the profit share.

The leaser's decision problem can be formulated as follows:

$$\max_{\alpha\ r} L = Q - \alpha Q - r\,, \tag{1}$$

where Q - *leaser's* profit.

To determine the optimal compensation, which should be offered to the *agent*, the *agent's* decision problem is to be considered. It can be formulated as follows:

$$\max_{e} A = r + \alpha Q - \frac{k}{2}e^2 - \alpha^2 p^2 \frac{a}{2}\,, \tag{2}$$

where a - the degree of risk aversion of the agent, and k - the rate of increase of the marginal cost of effort of the agent, e - the level of the effort exerted by the agent in case he accepts the terms of this contract, p – the probability of the lessee's acceptance of the terms of lease agreement.

The constraint (2) is considered to be the incentive constraint, which guarantees that the agent will always exert the effort providing the principal's profit maximization.

It is also implied that the participation constraint is satisfied, according to which the agent's profit is guaranteed to be not less than his reservation utility. This constraint guarantees that the agent accepts the contract.

$$r + \alpha Q - \frac{K}{2}e^2 - \frac{a}{2}\alpha^2 p^2 \geq B\ , \tag{3}$$

where B - reservation utility of the *agent*.

Hence, in order to form the optimal contract the problem (1) is to be sold under conditions (2) and (3).

This problem has been solved by means of quadratic programming realized with the help of the program MATLAB. In order to determine the volume of each leasing deal of the three alternatives considered, the sum of total lease payment has been calculated for each variant. Based on the results received, the most profitable deal-structuring variant has been revealed.

Time-consistency of the Core in Group Pursuit Games

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Keywords: *Group pursuit game, Cooperative game, Nash equilibrium, Core, Time-consistency*

In this paper a game of group pursuit in which players move on a plane with bounded velocities is studied. The game is supposed to be a nonzero-sum simple pursuit game between an evader and *m* pursuers acting independent of each other. The case of complete information is considered. This means that each player, choosing control variables at each time instant $t > 0$, knows the moment t and his own as well as all other players' positions.

A new approach for finding solution in these games for a case of two pursuers and one evader is proposed in [2] The key point of the work is to construct some cooperative solutions of the game and compare them with non-cooperative solutions such as Nash equilibria. It is important to give a reasonable answer to the question if cooperation is profitable in differential pursuit games or not. We consider all possible coalitions of the players in the game. For example, a pursuer promises some amount of the total payoff to the evader for cooperation with him. In that way, a cooperative game in characteristic function form is constructed. It is proved that in this game there exists the nonempty core for any initial positions of the players**.** However, in a dynamic game existence of the core at the initial moment of time is not sufficient to be accepted as a solution of the game. In the paper it is proved that the core in this game is time-consistent (see [1]).

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The Stability of Voting Games with Abstention

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Keywords: *Core, Nakamura Number, Simple games, Social decision systems, Stability*

The models of vote in this work are devoted to bill adoption contexts. In such a context, a bill is considered by the community. Assume for instance that the issue is a referendum on whether or not to amend the constitution. The constitution in force (say *y*) is called the status quo. A projected constitution (say *x*) is a candidate to the replacement of *y*. Enforcing the bill consists of adopting the replacement, while rejecting the bill consists of discarding the candidate *x*. So, there are two options at the collective decisions: bill adopted, or bill rejected.

Simple games, introduced by Von Neumann (1947) assume that a coalition is either decisive (otherwise called winning, in which case it holds the entire power of decision), or non winning (in which case it is absolutely powerless), there are no provisions for abstentions, and abstentions (if any occurs) have effects that are equivalent to votes against the bill.

In the relative majority rule, the collective decision enforces the bill if the set S_1 of voters who vote for it is in cardinality greater than the set S_2 of those who vote against. If S_2 is the set of voters who abstain, the condition for enforcing the bill can be described by the inequality $|S_1| > |N \setminus (S_1 \cup S_2)|$ (or $|S_1| > |S_2|$), where $|Sp|$ denotes the cardinality of Sp .

Social decision systems (Sds), also called voting games with abstension, owed to Rubinstein (1980), are a generalization of the relative majority rule where, in addition to the two voting options (for or against), the voter can also abstain. The casting of all individual votes realizes a partition N , $\{S_1, S_2, S_2\}$ where N , S_1 , S_2 , and S_2 are respectively the sets of all voters, the set of voters who are for the bill, the set of those who abstain and the set of those who are against. Here, decisiveness is described using the couple $(S_1;S_2)$. Hence, $(S_1;S_2)$ is winning for an Sds means that if each member of S_1 votes for a bill and each member of S_2 abstains, then the bill is enforced no matter what the other voters out of S_1 and S_2 do. This model generalizes the well known concept of simple game and in turn is generalized by the so-called game with multiple levels of approval introduced by Freixas and Zwicker (2003)

In this paper, we adopt the model of social decision systems (Sds). Our main preoccupation is the study of the core with emphasis on the characterization of its nonemptiness for every preference profile of voters (Recall that the core is the set of stable candidate, that is, the set of candidate that cannot be vetoed by any winning couple $(S_1;S_2)$, or winning coalition in the case of a simple game). Such a characterization for simple games was obtained by Nakamura (1979).

We obtain a necessary and sufficient condition for which the core of an Sds is non-empty regardless of the profile of individual preferences. We show that our result is a generalization of the Nakamura's theorem.

An Application of a Complex Business Game to Joint Top Engineers and Executive MBA Players: the Case of the Mirage Simulation with the French Corps des Mines and the ESSEC-Mannheim EMBA/WE

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Keywords: *Business games, Mixing up of engineers and EMBAs, Internationalization of simulation participants*

In 2008, I have used the Mirage business game which I developed with Raymond Gambini of Aix-Marseille 2 University (Universite de la Mediterranee), for the first time jointly with the finishing engineer-students of the Corps des Mines and the students in the ESSEC-Mannheim Executive MBA/WE. The Corps des Mines students are top class graduates of Ecole Polytechnique and of Ecole Normale Supérieure, the very top of the French scientificc education. ESSEC is a top French graduate school of business, the first to be accredited by AACSB outside of North America, with campuses in Paris and Singapore. In 2004 ESSEC and the Mannheim Business School joined forces to create the ESSEC-Mannheim European Executive MBA. Mirage is a complex general management business game in the textile industry located in the European Union, facing increasingly tough international competition, with key strategic questions of branding and positioning. Competing companies are listed on the stock exchange and have a variety of financial tools available, including industry consolidation by buy-outs of competitors. They can also make different choices of investments and of technologies. Over the course of the game economic regulations change, and to resist cheap foreign labour, they can enter subcontracting agreements among themselves and delocalize production, thus building different and specific profiles. They can manage different brands on boutique and mass retail circuits, and can decide to become pure distributors on these brands.

Mirage is regularly used on the internet with several hundred students in many games in parallel with common posting of economic performances and alliances through negociations. The novelty with the Corps des Mines/ESSEC-Mannheim EMBA 2008 simulation has been the making of joint teams and the making of “real” boards of directors for each team to which players had to respond. These boards are made of real bankers and company executives and directors, some currently active and some recently retired.

The paper will describe the setting of the simulation, the business model and the game itself, and the lessons learned from the mixing-up in the same teams of populations very different in their education, professional experience and group characteristics, the Corps des Mines being a closed body while the Executive MBA students come from a variety of jobs and companies.

The question behind this experience is the widening of business games to worldwide simulations with competitors in several countries and from different professional backgrounds. There is no technical difficulty in this, and I have run in the past common simulations between ESSEC and Duke University in the US for instance, but only on a small basis and key questions remain in curricula developments and in coordination which are at the core of the internationalization and of the “commonalization” of both graduate and executive business education, this meaning mixing up of complementary companies, such as teams coming jointly from companies producing and marketing products or services and their support companies, such as advertising agencies, bankers and the like.

Equilibrium in a Procedure with Arbitration Committee

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Keywords: *Player, Offer, Arbitration committee, Jurymen, Equilibrium*

Abstract: *We consider a non-cooperative zero-sum game. Two players L and M have a dispute. Player L makes an offer x, and player M – an offer y. If y is greater than or equal to x there is no conflict, and the players agree on a payoff equal to half of sum x and y. If, other wise, x is greater than y, the parties call in the arbitration committee. It consists of n jurymen. Equilibrium in the arbitration game among pure strategies is derived.*

We consider a non-cooperative zero-sum game. Two players L and M have a dispute. Player L makes an offer x, and player M – an offer y. If y is greater than or equals x there is no conflict, and the players agree on a payoff equal to half of sum x and y. If, other wise, x is greater than y, the parties present their offers to some arbitration committee. Each member after observing the offers x and y decides which offer must be selected. Suppose that they select an offer follows by the final-offer arbitration scheme (FOA), i.e. each arbitrator select an offer which is closer to his decision.

For example, suppose that the number of members is three. Let the solutions of the arbitrators are presented by random variables a_1, a_2, a_3 with continuous distribution function $F_i(a)$.

If the medians of all distributions F_i are equal m, than the equilibrium has the form

$$\begin{cases} x^* = m + \dfrac{1}{f_1(m) + f_2(m) + f_3(m)} \\ y^* = m - \dfrac{1}{f_1(m) + f_2(m) + f_3(m)} \end{cases}$$

where f_i – densities of the distribution functions F_i.

For instance, if a_1, a_2, a_3 have normal distribution with the same mean value m and different standard deviations σ_1, σ_2, σ_3 we obtain

$$\begin{cases} x^* = m + \dfrac{\sqrt{2\pi}}{\sum_{i=1}^{3} \dfrac{1}{\sigma_i}} \\ y^* = m - \dfrac{\sqrt{2\pi}}{\sum_{i=1}^{3} \dfrac{1}{\sigma_i}} \end{cases}.$$

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Formation of the Coalitional Structure in a Heterogeneous Population

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Keywords: *Coalitional equilibrium, Coalitional structure*

The present paper aims to study endogenous formation of coalitional structures in framework of the non-cooperative game theory. We assume that each individual of the population is characterized by some parameter (for instance, her location or bliss point). The continuous distribution over this parameter describes the whole population. We propose the following simple model of coalitions' formation. There is a large finite set of labels: “coalition 1”, “coalition 2”,…, “coalition M”. Each individual (player) chooses one of these labels and becomes a member of the corresponding coalition, or decides to abstain and stay alone. A given strategy profile determines the set of non-empty coalitions, the size and the strategy of each coalition from this set. We assume that the strategy is a point in the same parameter space. This point is determined depending on the distribution of coalition members’ parameters according to a certain rule (for instance, a median or mean rule). For each player, her payoff depends on two values: it increases in the size of the coalition including the player, and decreases in the distance between the individual parameter and the coalition strategy. For this game, we study Nash and coalitional equilibria and characterize corresponding coalitional structures.

There are two main streams in the literature related to endogenous formation of coalitional structures. One considers formation of jurisdictions (municipalities or regions) (Alesina, Spolaore (1997, 2003), Weber, Le Breton (2002), Haimanko, Le Breton, Weber (2002a,b)) by individuals located on some line or plain. Another stream relates to endogenous formation of political parties (Caplin, Nalebuff (1997), Ortuno-Ortin, Roemer (2000), Gomberg, Marhuenda, Ortuno-Ortin (2000, 2005)).

The following characters distinguish the present paper from the literature: continuous distribution of individuals in the space, no side payments, the individual payoff dependent on the coalition size and the distance between the individual bliss point and the coalition strategy.

Our main results are as follows. For n-dimensional Euclidian parameter space with a uniform distribution of individuals, there exist different types of Nash equilibrium (NE) structures, and we focus on the structures corresponding to the uniform rectangular grids. If any coalition corresponds to the rectangular parallelepiped with the edges parallel to the axis, only such grids determine NE coalitional structure. For these structures we consider several concepts of coalitional stability. The structure is stable with respect to splitting if there exists no new coalition that is a proper subset of some coalition in the structure and provides greater payoffs to all its members. The structure is stable with respect to local unification if there is no new coalition that is a union of several neighbor coalitions and provides greater payoffs to all its members. We obtain necessary and sufficient conditions of stability with respect to some types of unions and splits. We show that existence of non-trivial stable structures crucially depends on relation between the space dimension n and the degree k of the main term in the payoff function Taylor expenditure in the distance between the individual bliss point and the coalition strategy. If then the only possible stable structure is atomic (nobody joins any coalition) or the global union (everybody joins one coalition). The first variant takes place if the coefficient before the main term (the non-conformity coefficient) is larger than some threshold, and the second variant occurs if the coefficient is less than this threshold. For we determine the interval for the non-conformity coefficient where the non-trivial stable structures exist.

Section 3 provides more complete results for the one-dimensional parameter space – interval. We show that for any regular NE (with different strategies of different coalitions) the coalitional structure is a partition of the space into intervals corresponding to different coalitions or including abstainers.

We call NE a weak coalitional equilibrium (WCE) if there is no new coalition that provides greater payoffs to all its members. We determine WCE for several types of payoff functions and distributions. We assume that the payoff linearly increases in the coalition size and either linearly or quadratically decreases in the distance between the coalition strategy and the individual bliss point. For a linear payoff function the WCE is typically unique and corresponds to some trivial structure: if the non-conformity coefficient is less than 2 then this is a global union, and if the coefficient is more than 2 then this is an atomic structure. For a quadratic payoff function we limit our study with the case of the uniform distribution. We show that for any non–conformity coefficient below some threshold the only WCE is the global union. Above this threshold the number of WCE, the minimum and the average number of coalitions in the WCE increase in the non-conformity coefficient.

In conclusion we discuss some reasons for different political structures and different number of political parties in the modern world.

The Extension of the Dutta – Ray Solution to a Subclass of Balanced TU Games

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Keywords: *Dutta-Ray solution, Consistency, Cooperative TU game*

Abstract: *The egalitarian solution for the class of convex TU games was defined by Dutta and Ray (1989) and axiomatized by Dutta 1990.An extension of this (DR) solution – the egalitarian split-off set (ESOS) – to the class of all TU games was proposed by Branzei et al. (2006). The solution is the result of Dutta's algorithm applied to arbitrary TU games. For non-convex games the solution may be multi-valued even for two-person games A subclass of balanced games containing the convex games whose Dutta—Ray solutions are single-valued and contained in the core is found and characterized by constrained egalitarianism, the Davis—Maschler consistency and continuity.*

The egalitarian solution for the class of convex TU games was defined by Dutta and Ray (1989) and axiomatized by Dutta 1990. An extension of this (DR) solution – the egalitarian split-off set (ESOS) – to the class of all TU games was proposed by Branzei et al. (2006). The solution is the result of Dutta's algorithm applied to arbitrary TU games. For non-convex games the solution may be multi-valued even for two-person games. Moreover, for two-person subadditive games it loses the egalitarian traits giving all the gain to the "rich" player with the greater value of individual characteristic function value.

Thus, in order to obtain a convincing extension of the Dutta-Ray solution to a wider class of TU games for two-person games this class should consist of super-additive games. If the extension from two-peson games to arbitrary player sets games is fulfiled with the help of the Davis-Maschler consistency, then the extended class should be a subclass of balanced games, and the extended solution should be a subset of the core.

Such a subclasses of balanced TU games do exist. For example, if x is the Dutta-Ray solution of a convex game (N,v) then x Lorenz dominates all payoff vectors from the core of (N,v') , where v' is an arbitrary characteristic function satisfying the relations

If $x(S)>v(S)$, then $x(S)>$ or $= v'(S)$.

A subclass $G(bc)$ of the class $G(b)$ of balanced games consisting of the games whose cores contain the vectors which Lorenz dominate all other vectors from the core, is considered. For each game from this class the ESOS turns out to be single-valued and belongs to the core, so for this class we may put ESOS=DR.

The class $G(bc)$ is essentially wider than the class of convex games, because given a game (N,v) from $G(bc)$, any decreases of characteristic function values of coalitions, having no influence on its DR solution, do not lead the game out of the class. In fact, it is easy to note that the Lorenz maximal vector x from the core will contain the property to be maximal with decreasing of the values $v(S)$ for coalitions S such that $x(S)>\mathrm{v}(S)$.

Each sublass of $G(bc)$ with a fixed value of the grand coalition can be represented as the union in all payoff vectors x of the collections $G(x)$ of games whose DR solutions equal x:

$G(x)=\{(N,v)|\mathrm{DR}(N,v)=x\}$.

A characterization of the collections $G(x)$ is given implying the following properties of the DR solution for the class $G(bc)$:

Theorem 1. The DR solution is coalitionally monotonic on the class $G(bc)$.

Theorem 2. The DR solution satisfies the Davis-Maschler consistency on the class $G(bc)$.

However, contrary to convex games, the DR solution on the class $G(bc)$ does not satisfy the converse consistency.

Another monotonicity property – population monotonicity – cannot be defined for the DR solution on the class $G(bc)$, because subgames of a game (N,v) from $G(bc)$ may not belong to this class.

The absence of the converse consistency does not permit to extend Dutta's characterization of the egalitarian solution to the class $G(bc)$. In fact, Dutta's proof of the uniqueness of the solution satisfying CE and the Davis-Maschler consistency used the converse consistency of the DR solution. Possibly, such an axiomatization of the DR solution on the class $G(bc)$ can be proved, but its existence is an open problem. Thus, the following axiomatic characterization of the DR solution for the class $G(bc)$ has been obtained by adding one more – continuity – axiom to the mentioned two ones:

Theorem 3. On the class $G(bc)$ the Dutta-Ray solution isa unique value that satisfies DM consistency, constrained egalitarianism, and continuity.

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Cooperative Game-theoretic Mechanism Design for Optimal Resource Use

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Economic analysis no longer treats the economic system as given since the appearance of Leonid Hurwicz's pioneering work on mechanism design. The term "design" stresses that the structure of the economic system is to be regarded as an unknown. New mechanisms are like synthetic chemicals: even if not usable for practical purposes, they can be studied in a pure form and so contribute to the understanding of the difficulties and potentialities of design. The design point of view enlarges our vision and helps economics avoid a narrow focus on existing institutions. The failure of the market to provide an effective mechanism for optimal resource use will arise if there exist imperfect market structure, externalities, imperfect information or public goods. These phenomena are prevalent in the current global economy. As a result not only inefficient outcomes like over-extraction of natural resources had appeared but gravely detrimental events like catastrophe-bound industrial pollution had also emerged.

Cooperative games suggest the possibility of socially optimal and group efficient solutions to decision problems involving strategic actions. This lecture focuses on cooperative game-theoretic design of mechanisms for optimal resource use. Since resource use is often a dynamic process we concentrate on mechanism designs involving an intertemporal framework. In such a framework, the stringent condition of dynamic (subgame) consistency is required for a dynamically stable cooperation mechanism. In particular the mechanism's optimality principle has to remain optimal at any time within the cooperation duration given any feasible state brought about by prior optimal behaviors. Crucial features that are essential for a successful mechanism are developed. In particular, individual rationality, group optimality, dynamic consistency, distribution procedures, budget balance, financing, incentives to cooperate and practicable institutional arrangements are incorporated in the design. Finally, cooperative game-theoretic mechanism design is used as an alternative approach in establishing the foundation for an effective policy menu to tackle sub-optimal resource use problems which the conventional market mechanism fails to resolve.

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Time Consistency in Cooperative Differential Games: A Tutorial

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Keywords: *Time consistency, Differential games, Sustainability of cooperation*

How can a cooperative agreement made at the start of a dynamic game can be sustained over time? Early work has avoided this question by supposing that the players sign binding agreements. This assumption is hard to accept from a theoretical perspective, and a practical one as well. Conceptually, there is no reason to believe that rational players would stick to an agreement if they can achieve a better outcome by abandoning, no matter what they have announced before. At an empirical level, it suffices to look at the number of disputes (between spouses, business partners, countries, etc.) in the courts to convince ourselves that binding agreements are not so binding. Scholars in dynamic games have followed different lines of thoughts to answer the question. This tutorial reviews one of them, namely time consistency, a concept which has also been termed dynamic individual rationality, sustainability, dynamic stability, agreeability, or acceptability.

Strong Nash Equilibrium in Stochastic game

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Keywords: *Strong Nash equilibrium, Stochastic game*

Abstract: *A stochastic game with infinite duration is considered. It is supposed, that transition probabilities in discrete case depends on the players' controls so that in limiting case one obtains stochastic Ito's process with controllable coefficients. A characterization of strong Nash equilibrium is obtained as a system of stochastic Bellman equations.*

A stochastic discrete game G with n players, infinite set of sets Γ^k with step length Δt is considered. Any set is a game of normal form $\Gamma^k = \left\langle N, \{Q_i\}_{i=1}^n, \{g_i(x^k,\cdot)\}_{i=1}^n \right\rangle$, where a variable x^k describes game dynamics. Suppose that in every step a value o x^k could be increased on amount $\Delta h = \sigma\sqrt{\Delta t}$ with probability $p_1 = \frac{1}{2}\left[1 + \frac{\mu(x,q)}{\sigma(x)}\sqrt{\Delta t}\right]$ or could be decreased on the same amount with probability $p_2 = \frac{1}{2}\left[1 - \frac{\mu(x,q)}{\sigma(x)}\sqrt{\Delta t}\right]$, $q \in Q = \prod Q_i$. It was proofed, that strong Nash equilibrium can be characterized as follows.

Theorem. For any $\Delta t > 0$ and $\sigma > 0$ a couple of strategies $q^*(x) = (q_1^*(x),...,q_n^*(x))$ provides a strong Nash equilibrium to the game G, if for any coalition $S \subseteq N$, $S \neq \varnothing$ there exist suitably smooth functions V_S, satisfying the following differential equations:

$$rV_S = \frac{1}{2}\sigma V_S'' + \max_{q_{i_1},...,q_{i_s}\ i_1,...,i_s \in S} \Big\{ \mu(x, q_1^*(x),...,q_{i_1-1}^*(x), q_{i_1}, q_{i_1+1}^*(x),...,q_n^*(x))V_S'(x) +$$

$$+ \sum_{i_1,...,i_s \in S} g_i[x, q_1^*(x),...,q_{i_1-1}^*(x), q_{i_1}, q_{i_1+1}^*(x),...,q_n^*(x)] \Big\}$$

where $g_S[t,x,q_1(x),q_2(x),...,q_n(x)] = \sum_{i_1,...,i_s \in S} g_i[t,x,q_1(x),q_2(x),...,q_n(x)]$.

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