Genetic Search Algorithms to Fuzzy Multi-objective Games: a Mathematica Implementation

ファジーマルチ客観的なゲームへの遺伝検索アルゴリズム：

Mathematica インプリメンテーション。

André A. Keller

Université de Lille 1 Sciences et Technologies, France

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ACS Session: Computer Science Algorithms I

Plenary Lecture 19
3rd Day: 09:30-10:00, Room B'

- I. Fuzzy bimatrix games with genetic search algorithms

  (1. Games' equivalence theorems, 2. Bimatrix games with fuzzy goals, 3. Genetic search algorithms to the Nash equilibrium solutions)

- II. Multi-objective fuzzy matrix games with genetic search algorithms

  (1. Nishizaki-Sakawa model, 2. Numerical example with Mathematica, 3. GENOCOP III software)

Outline

- I. Genetic algorithms to searching a Stackelberg equilibrium in bilevel optimization with GENOCOP III

- II. Genetic algorithms to the Pareto front determination in multi-objective optimization with NSGA II

Part I

Genetic algorithms to searching a Stackelberg equilibrium in bilevel optimization with GENOCOP III

1.1 Bilevel optimization equivalence

$$\min_{\mathbf{x}} \quad F(\mathbf{x},\mathbf{y}), \quad \mathbf{x} \in \mathbb{R}^{n_1}$$

subject to:

$$G(\mathbf{x},\mathbf{y}) \le 0,$$

$$\min_{\mathbf{y}} f(\mathbf{x},\mathbf{y}), \quad \mathbf{y} \in \mathbb{R}^{n_2}$$

subject to: $g(\mathbf{x},\mathbf{y}) \le 0,$

$$\min_{x,y,\vec{\lambda},\alpha,\beta} \quad F(x,y), \quad x \in \mathbb{R}$$

subject to:

$$\nabla_y \mathcal{L}(x_0, y, \vec{\lambda}, \beta) = 0,$$

$$\lambda_i g_i(x,y) = 0, \; i = 1,4$$

$$g_i(x,y) \le 0, \; i = 1,4$$

$$\alpha\, x = 0, \; \beta\, y = 0$$

$$x \ge 0, \; y \ge 0, \; \lambda_i \ge 0, \; i = 1,4, \; \alpha \ge 0, \; \beta \ge 0.$$

I.2 Bard's test-example (1/3)

$$\min_{x} \; F(x, y) = x - 4y, \quad x \in \mathbb{R}$$

subject to:

$$\min_{y} f(y) = y, \quad y \in \mathbb{R}$$

$$\begin{aligned}
\text{subject to:} \quad g_1(x, y) &\equiv -x - y + 3 \leq 0, \\
g_2(x, y) &\equiv -2x + y \leq 0, \\
g_3(x, y) &\equiv 2x + y - 12 \leq 0, \\
g_4(x, y) &\equiv -3x + 2y + 4 \leq 0, \\
x \geq 0, &\; y \geq 0.
\end{aligned}$$

I.2 Bard's test-example: equivalent single level optimization problem (2/3)

$$\min_{x,y,\vec{\lambda},\alpha,\beta} \quad F(x,y), \quad x \in \mathbb{R}$$

subject to:

$$\nabla_y \mathcal{L}(x_0, y, \vec{\lambda}, \beta) = 0,$$

$$\lambda_i g_i(x,y) = 0, \; i = 1,4$$

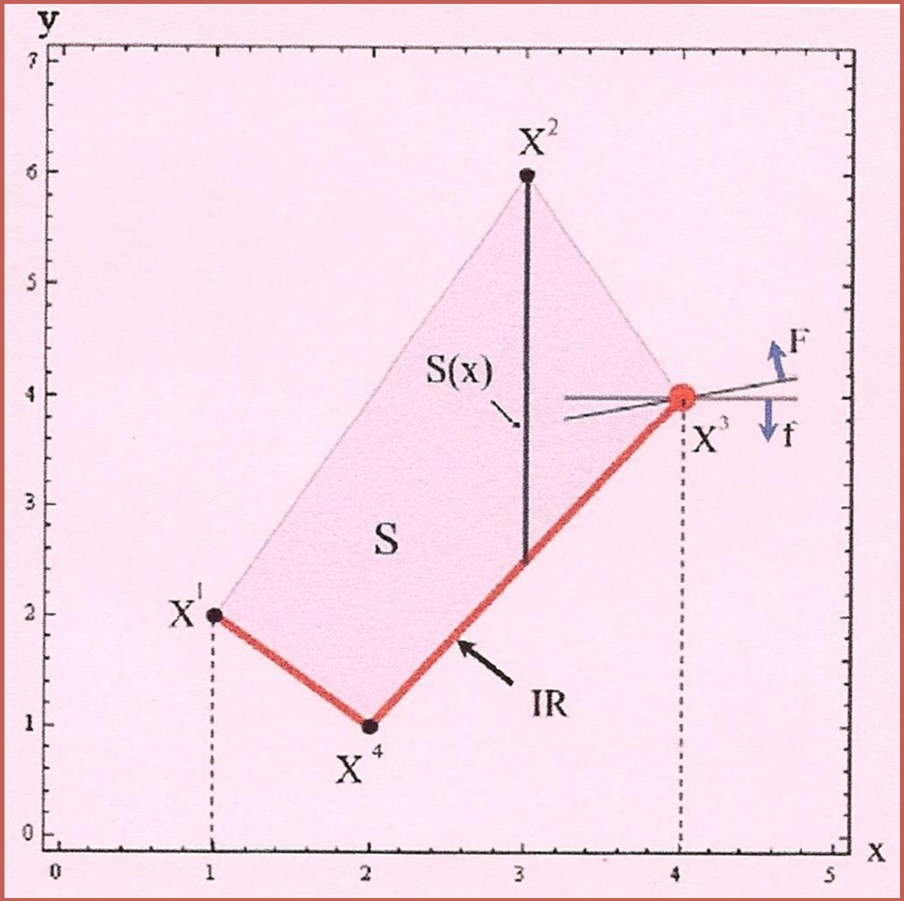
$$g_i(x,y) \leq 0, \; i = 1,4$$

$$\alpha\, x = 0, \; \beta\, y = 0$$

$$x \geq 0, \; y \geq 0, \; \lambda_i \geq 0, \; i = 1,4, \; \alpha \geq 0, \; \beta \geq 0.$$

$$\mathcal{L}(x_0, y, \vec{\lambda}, \beta) \equiv f(y) + \sum_{i=1}^{4} \lambda_i g_i(x_0, y) - \beta y,$$

I.2 Bard's test-example (3/3)



I.3 GENOCOP III software: main features

- GENOCOP is for Genetic algorithm for Numerical Optimization of Constrained Problems. It has been developed by Z. Michalewicz at UNC Charlotte/USA.
- The package has been implemented in a DOS system by using the compiler ACC from Absoft C/C++. It is connected to Mathematica 7 for initial preparation and further analysis.
- The system initially creates two types of populations of potential solutions. The feasibilty of the points is maintained through specific operators.
- Learning parameters are: population size, number of generations, selection pressure, probability of replacement of unfeasible search vector, mode of choosing reference (fully feasible) at random or using the selection pressure.

Part II

Genetic algorithms to the Pareto front determination in multi-objective optimization with NSGA II

II.1 Multi-objective optimization problem

$$\min_{\mathbf{x}} \ \mathbf{f}(\mathbf{x}) = \left(f_1(\mathbf{x}), \ldots, f_k(\mathbf{x})\right), \quad \mathbf{x} \in \mathbb{R}^n$$

subject to:

$$g_i(\mathbf{x}) \leq 0, \ i = 1, \ldots, p,$$

$$h_i(\mathbf{x}) = 0, i = p+1, \ldots, m,$$

$$\mathbf{x} \in [\mathbf{x}_l, \mathbf{x}_u].$$

II.2 Deb 's test-example: connected Pareto-optimal front (1/2)

$$\min_{\mathbf{x}} \; f_1(\mathbf{x}) = x_1, \quad \mathbf{x} \in \mathbb{R}^2$$

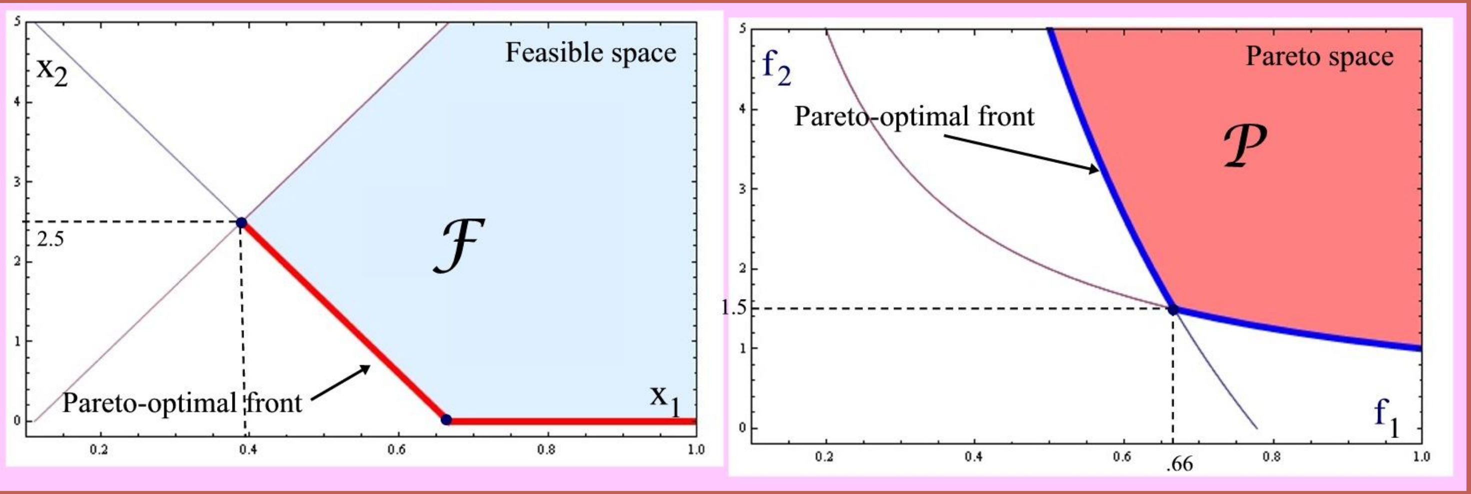
$$\min_{\mathbf{x}} \; f_2(\mathbf{x}) = \frac{1 + x_2}{x_1},$$

subject to:

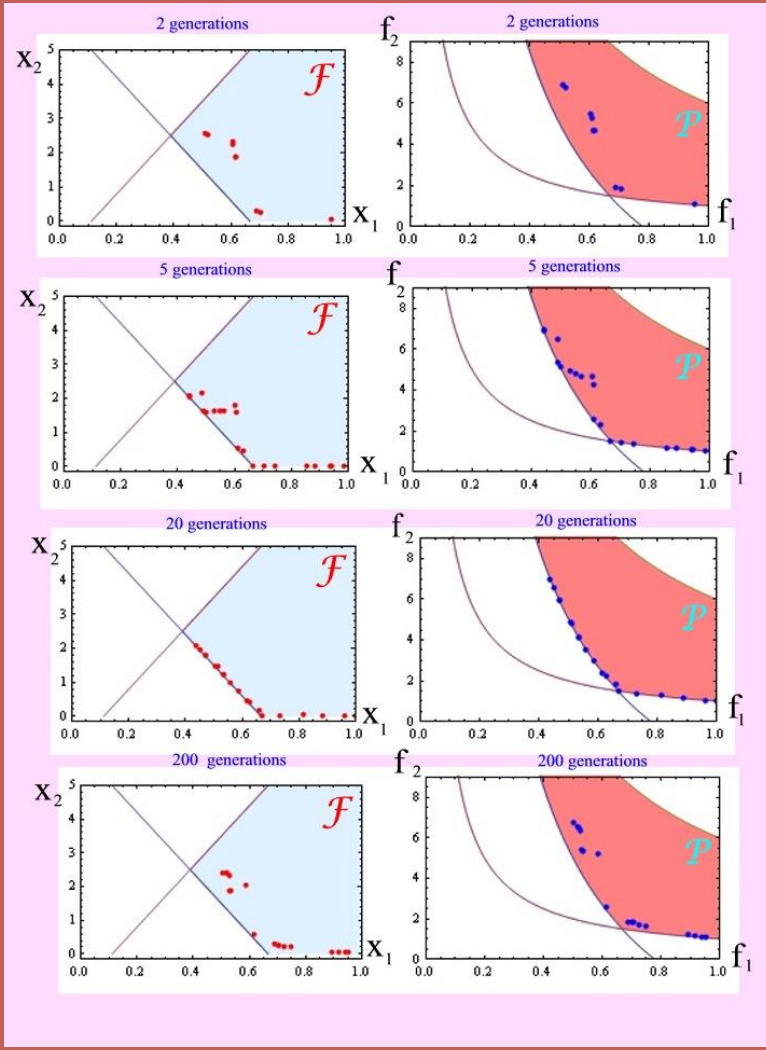
$$g_1(\mathbf{x}) \equiv 6 - 9x_1 - x_2 \leq 0,$$

$$g_2(\mathbf{x}) \equiv 1 - 9x_1 + x_2 \leq 0,$$

$$x_1 \in [.1, 1], \; x_2 \in [0, 5].$$



II.2 Deb's test-example (2/2)



II.3 Tanaka 's test-example : disconnected Pareto-optimal sets (1/2)

$$\min_{\mathbf{x}} \quad f_1(\mathbf{x}) = x_1, \quad \mathbf{x} \in \mathbb{R}^2$$

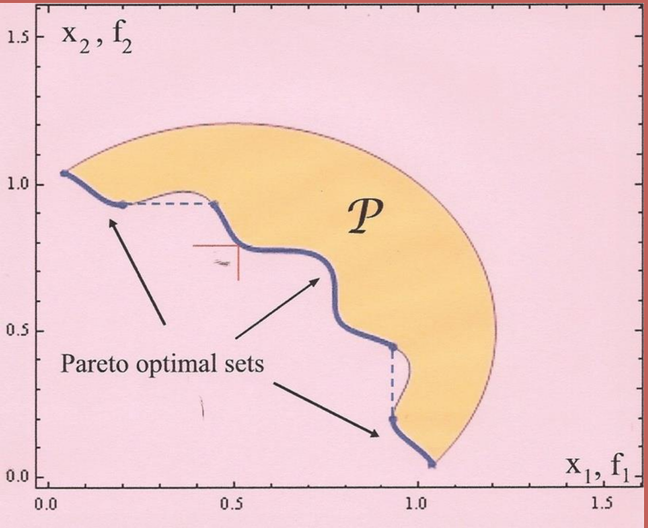
$$\min_{\mathbf{x}} \quad f_2(\mathbf{x}) = x_2,$$

subject to:

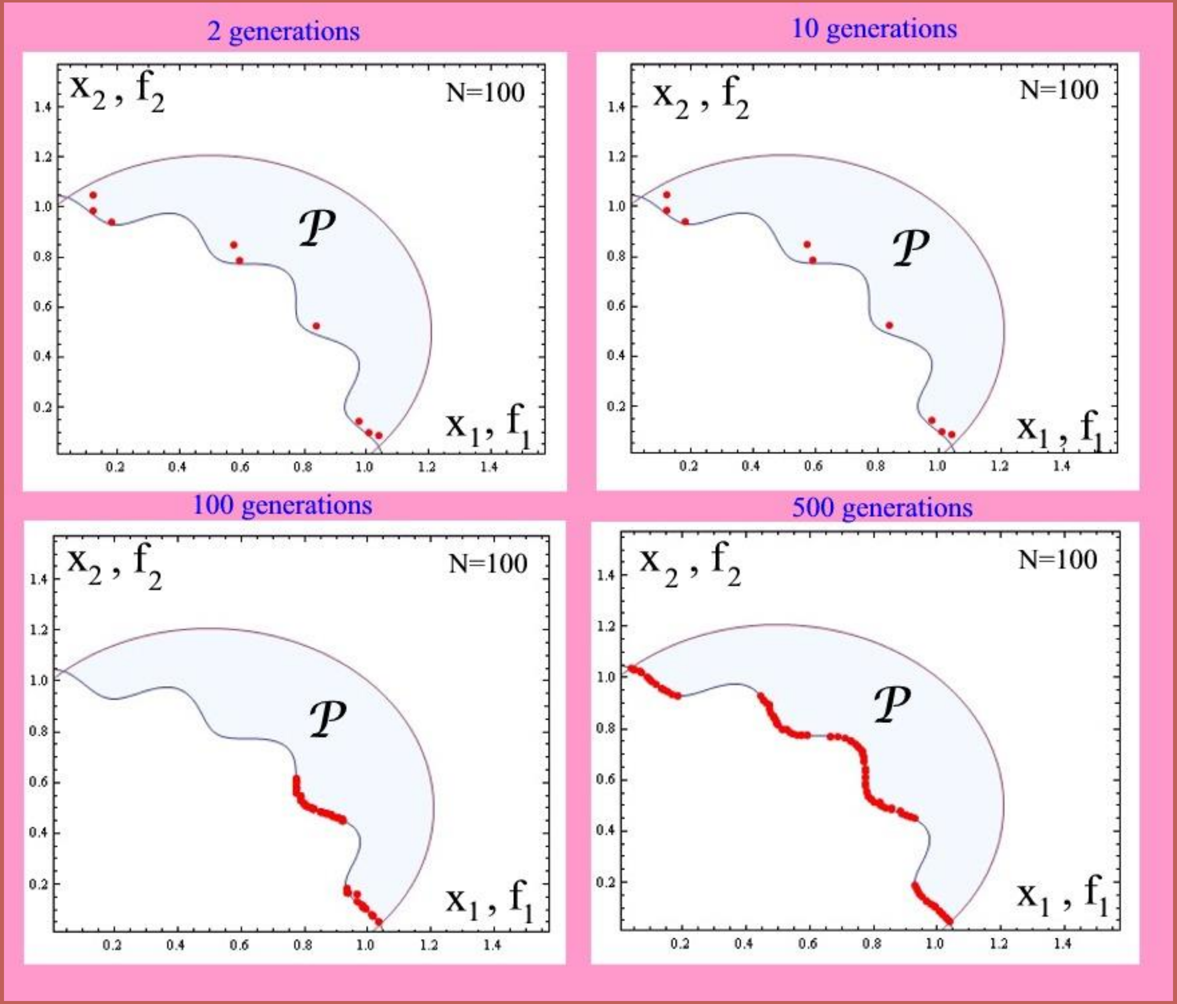
$$g_1(\mathbf{x}) \equiv x_1^2 + x_2^2 - 1 - .1\cos(16 \arctan \frac{x_1}{x_2}) \geq 0,$$

$$g_2(\mathbf{x}) \equiv (x_1 - .5)^2 + (x_2 - .5)^2 - .5 \leq 0,$$

$$x_1,\ x_2 \in [0, \pi].$$



II.3 Tanaka 's test example : disconnected Pareto-optimal sets (2/2)



II.4 NSGA II sofware: main features

- NSGA is for Nondominated Sorting Genetic Algorithm, by Deb et al. at Kanpur Genetic Algorithms Laboratory. It is a fast and elitist multi-objective evolutionary algorithm, sorting the population in different fronts by using the non-domination order relation.
- For this study, the package has been implemented in a DOS system by using the compiler ACC from Absoft C/C++. It is connected to Mathematica 7 for further results.
- The system uses a Pareto ranking procedure and incorporates a fitness sharing. To form the next generation, the algorithm combines the current population and its offspring with bimodal crossover and polynomial operators.
- Learning parameters are: population size, number of generations (<5000), crossover and mutation probability, the distribution index for crossover, a random generator seed parameter.

Thank You for your attention.

あなたの注意をありがとうございます。

芸者

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